Equilibrium Counterfactuals*

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Abstract

We incorporate structural modellers into the economy they model. Using the traditional moment-matching method, they ignore policy feedback and estimate parameters using a structural model that treats policy changes as zero probability (or exogenous) "counterfactuals." Estimation bias occurs since the economy's actual agents, in contrast to model agents, understand policy changes are positive probability endogenous events guided by the modellers. We characterize equilibrium bias. Depending on technologies, downward, upward, or sign bias occurs. Potential bias magnitudes are illustrated by calibrating the Leland (1994) model to the Tax Cuts and Jobs Act of 2017. Regarding parameter identification, we show the traditional structural identifying assumption, constant moment partial derivative sign, is incorrect for economies with endogenous policy optimization: The correct identifying assumption is constant moment total derivative sign accounting for estimation-policy feedback. Under this assumption, model agent expectations can be updated iteratively until the modellers' policy advice converges to agent expectations, with bias vanishing.

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1. Introduction

The strength of structural microeconometric methods relative to quasi-experimental methods is their (potential) ability to overcome the rational expectations critique of Lucas (1976) who described pitfalls in extrapolating econometric estimates across policy environments. Keane and Wolpin (1997) write, "The desire to better forecast behavior after regime changes was a key impetus to structural estimation." Rust (2013) writes, "The reason why we want to restrict attention to structural models is well understood: econometric policy evaluation and forecasting is either impossible or highly unreliable using non-structural or quasi-structural models. This is the point of the famous Lucas critique (1976)." Blundell (2017) writes, "By specifying the parameters that describe the preferences and constraints of the decision-making process, structural models deliver counterfactual predictions. The ability to provide policy counterfactuals sets them apart from reduced-form models."

The starting point for this paper is to note that, intent notwithstanding, existing structural microeconometric methods will generally *violate* rational expectations when the structural models serve their intended function of rigorously informing policy decisions. To see the violation, recall that, as argued by Sargent (2005), under rational expectations, "There is a communism of models. All agents inside the model, the econometrician, and God share the same model."[1] Hansen and Sargent (2010) write, "The rational expectations hypothesis imposes a communism of models: the people being modeled know the model. This makes the economic analyst, the policy maker, and the agents being modeled all share the same model, i.e., the same probability distribution over sequences of outcomes." Notice, the internal consistency inherent in communism of models is not simply a matter of aesthetics. Rather, the imposition of "communism" is what allows the econometrician to avoid the type of policy pitfalls that so troubled Lucas (1976) in the first place, such as illusory free-lunches arising from government exploiting systematic divergences between the beliefs of modeled agents versus the actual distribution of variables under prescribed policy.

With the preceding discussion in mind, consider that in specifying the decision problem of agents inside her model, the structural microeconometrician must specify government policy. Critically, it is customary to parameterize models according to status-quo policies (or to specify policy as an

[1] Quoted in Evans and Honkapohja (2005).

exogenous process). This standard practice violates rational expectations since the agents inside the models are treated as being ignorant of the existence of econometricians and the role their parameter estimates play in informing endogenous policy choices. That is, the agents inside models are treated as being ignorant of future endogenous policy changes despite the goal of the econometrician being to inform those endogenous policy decisions.

What are the implications of such violations of rational expectations in microeconometrics, and what can be done about them operationally? To address these questions, we consider an economy in which "real-world" agents with rational expectations are placed alongside a structural microeconometrician who will give policy advice. That is, we develop a model that incorporates the structural modeler, in the spirit of the communism of models of Sargent (2005). The real-world agents are privately endowed with a policy-invariant parameter.[2] Knowledge of this parameter would be sufficient for the government to set policy at first-best. The econometrician will observe an empirical moment derived from agent actions which will serve as the basis for her parameter inference.

When the model opens, the government policy variable is set at a pre-determined "status quo" value, denoted γ_0.[3] Nature then draws the unknown parameter u from the real line. There is a continuum of rational real-world agents who privately observe the parameter and choose their actions non-cooperatively. The econometrician then observes a moment m that satisfies the *standard moment monotonicity condition*: The partial derivative $\partial m/\partial u$ has constant sign.[4] The econometrician then matches the model-implied moment with the observed empirical moment in order to draw an inference $\widehat{u}$. Finally, with positive probability the government will subsequently enjoy discretion to move the policy variable away from γ_0 based upon the econometrician's report of $\widehat{u}$.

We begin by offering a catalog of the nature of biases that arise if real-world agents have rational expectations while the structural model violates rational expectations by treating changes in policy away from γ_0 as zero probability ("counterfactual") events.[5] The real-world agents rationally anticipate the econometrician's inference $\widehat{u}(u)$. More generally, these agents rationally anticipate the "equilibrium" function $\widehat{u}(\cdot)$ determining the econometrician's inference for each possible realized

[2] Idiosyncratic parameters can be, say, multiples of a common aggregate parameter.

[3] As discussed below, γ_0 can be policy in some state under exogenous Markovian policy.

[4] See Gallant and Tauchen (1996) and Kahn and Whited (2017) for discussion.

[5] Our argument also applies if γ_0 is an exogenous element of a Markovian policy vector.

value u.[6] Agents then correctly anticipate the policy the government will implement on the date it enjoys policy discretion, denoted $g[\widehat{u}(u)]$.

As shown, the failure to impose rational expectations results in $\widehat{u}(u) \neq u$ at all points except the one possible realization of the unknown parameter, call it u_0, that would justify the government maintaining the status quo γ_0. Intuitively, in this exceptional (measure zero) case, the econometrician will not violate rational expectations since parameterizing the structural model as if the status quo will be implemented even when the government enjoys policy discretion is actually correct here.

The nature of bias depends upon the properties of the moment function m and the government policy function g mapping reports $\widehat{u}$ to discretionary policy. The central insight is as follows. If real-world agents have rational expectations, the empirical moment first varies directly with changes in the parameter u. This *direct effect*, m_u, is accounted for by the structural model. However, the real-world moment also varies indirectly due to the rational expectations of agents of feedback from the parameter inference ($\widehat{u}$) to discretionary government policy ($g(\widehat{u})$). Thus, the real-world empirical moment can be expressed as $m[u, g(\widehat{u}(u))]$. It is the *indirect effect* arising from joint estimation and policy control, $m_g g' \widehat{u}'$, that is generally omitted in structural microeconometric estimation. Phrased differently, the policy expectations of agents are not constants and not exogenous processes but are instead functions of the deep parameters being estimated.

There are three cases to consider. In the first case, the sign of the indirect effect is opposite to that of the direct effect. Here, the estimated parameter overshoots for $u < u_0$ and then undershoots for $u > u_0$ (and recall, u_0 justifies the status quo policy γ_0). Intuitively, the modeler incorrectly treats small observed changes in the empirical moment to small changes in u because she here fails to account for the countervailing indirect effect. In the second case, the indirect effect is small in absolute value and has the same sign as the direct effect. Here, the estimated parameter undershoots for $u < u_0$ and overshoots for $u > u_0$. Intuitively, the modeler incorrectly treats large observed changes in the empirical moment to large changes in u because she here fails to account for the amplifying indirect effect. In the third case, the indirect effect is large in absolute value and has the same sign as the direct effect. Here, the estimated parameter actually decreases with the true parameter, and it possible for an equilibrium to arise where the estimated parameter always has the wrong sign. The subtle intuition for this case is provided in the body of the paper.

[6] In this "equilibrium" real-world agents are rational maximizers but the structural econometrician makes errors.

We illustrate the potential quantitative significance of these effects by considering an econometrician whose objective is to infer bankruptcy costs using the canonical structural model of Leland (1994). In particular, we consider the recent cut in the corporate income tax rate implemented under the Tax Cuts and Jobs Act of 2017. Here the structural microeconometrician backs out implied bankruptcy costs from observed values of corporate interest coverage ratios. By assumption, the econometrician knows the underlying real technology but fails to impose the assumption of rational expectations on the part of firms inside the model. In our calibrated example, this leads to an eight-fold overstatement of bankruptcy costs. Intuitively, firms rationally anticipate a tax cut and thus choose low leverage in light of the low value of future debt tax shields. Neglecting this fact, the econometrician mistakenly infers that the low leverage stems from extremely high bankruptcy costs.

After characterizing bias, we then describe how, under technical conditions, unbiased parameter inference and first-best government policy can be achieved. Essentially, the econometrician must insist that the beliefs of the agents inside her model are consistent with the policy advice she will give. This involves recognizing that the observed empirical moment will vary directly with the unknown parameter u and also with agents' rational expectation of government policy changes in light of (correct) inference of the structural parameter. As we show, such internal consistency can be achieved via iteration on policy advice until a fixed-point is found where the policy advice supports the parameter inference, and vice-versa.

Turning to the technical conditions, we show the standard moment monotonicity condition, which focuses on partial derivatives of moments, is neither necessary nor sufficient for structural parameter identification when estimation is actively informing policy rather than passive. Rather, we propose total derivatives, cum policy feedback, as the correct moment selection criterion in the context of joint estimation and control exercises. That is, moment selection criteria are altered as one moves from passive to active estimation. For example, a moment that is informative (uninformative) under passive estimation may be uninformative (informative) under active estimation.

In broad details, the model applies to any entity attempting to infer some parameter for a group of agents, provided that the agents are forward-looking rational maximizers, and provided that the entity will act based upon its parameter inference with positive probability. For example, firms might be interested in estimating policy-invariant consumer parameters with the goal of evaluating alternative pricing strategies, product characteristics, or marketing expenditures.[7] Our expositional focus on structural parameter inference for the purpose of government policy setting is simply to fix ideas.

The model is intended to be forward-looking in terms of its application. That is, it might be plausibly argued that in many settings the link between econometric evidence and decision-making is weak at the present time. However, most economists strongly advocate a move toward systematic evidence-based decision-making by governments and firms. Moreover, it is clear that governments notwithstanding, modern firms are increasingly moving toward data-driven decision-making. Our framework can be seen as capturing the types of problems of econometric inference that will intensify as systematic data-driven decision-making becomes commonplace, and as real-world agents understand this to be taking place.

We turn now to other related literature. At core, our argument is related to the seminal paper by Hurwicz (1962) which offers an early formal definition of *structure* in econometrics research. According to Hurwicz, an equation can be said to be structural if it is invariant over the "domain of modifications anticipated." He writes:

> The concept of structure is relative to the domain of modifications anticipated. In particular, the structure is not necessarily defined for every domain W. Hence a certain equation of a system may be in structural form relative to some W' but not relative to W''. If two individuals differ with regard to modifications they are willing to consider, they will probably differ with regard to the relations accepted as structural.

The essence of our argument is that if real-world agents have rational expectations, and if the structural analysis is actually policy-relevant, the empirical moments targeted by modellers are not invariant over the domain of policy modifications considered by modellers.

Technical challenges inherent to problems of joint estimation and control under rational expectations have largely escaped the attention of microeconometricians. This is understandable given that the first-stage priority in this research programme has been to reduce the dimensionality of complex models designed to capture the granular details of public welfare programs and the like. By way of contrast, in macroeconometrics, Sims (1980, 1982) and Hansen and Sargent (1991) engaged

[7] See Chintagunta, Erdem, Rossi and Wedel (2006) for a survey of structural methods in marketing.

in a vigorous debate regarding the correct interpretation and utilization of vector autoregressions in guiding monetary policy. Here the distinguishing feature of rational expectations is insistence upon theory-founded cross-equation restrictions being imposed on vector autoregressions.

In the spirit of our paper, Sargent (1987) sketched the existence of problems inherent in joint estimation and control under rational expectations: "There is a logical difficulty in using a rational expectations model to give advice, stemming from the self-referential aspect of the model that threatens to absorb the economic adviser into the model... That simultaneity is the source of the logical difficulties in using rational expectations models to give advice about government policy." Striking a pessimistic note, Sargent (1984) states, "I am unaware of an alternative approach to Sims or to rational expectations econometrics that avoids these contradictions and tensions." These philosophical and logical difficulties apparently led Sargent to shy away from the use of macroeconometric models for the purpose of informing policy decisions. For example, Sargent (1998) states, "That's a hard problem. I don't make policy recommendations."[8] Importantly, at least in terms of structural microeconometrics, potential paradoxes arising from joint estimation and control do not appear to be insurmountable.

Gourieroux and Monfort (1995) provide an overview of structural methods. Adda and Cooper (2003) provide numerous applications of structural methods including firm-level labor and capital demand. Keane and Wolpin (2002) develop a granular structural model of public assistance programs. Keane and Wolpin (1997), Rust (2013), Wolpin (2013), Blundell (2017) provide overviews of structural microeconometric applications in labor economics, public finance, and industrial organization, amongst others. For examples of structural methods in corporate finance and banking see Gomes (2001), Moyen (2005), Hennessy and Whited (2005), and Egan, Hortascu and Matvos (2017).

Closest to the present paper is work by Chemla and Hennessy (2019) showing that an analogous problem of bias arises when quasi-experimental evidence is used to inform endogenous policy decisions. Arguably, the present paper's critique is more problematic in that it is internal, taking models and agent rationality seriously, a goal shared by many structural microeconometricians. Another important difference is that within the logic of a structural model, bias characterization is simpler. Finally, the present paper offers a feasible algorithm for avoiding bias and achieving

[8] Quoted in Sent (1998).

first-best policy, again within the logic of a structural model. Despite these differences, the two papers share the message that the econometric tool-kit changes fundamentally as one moves from passive to active policy-relevant estimation.

The rest of the paper is as follows. Section 2 describes the economic setting. Section 3 characterizes the nature of bias if the econometrician fails to impose rational expectations. Section 4 shows how, under technical conditions, unbiased parameter inference and first-best government policy can be achieved through a fully-consistent application of rational expectations. In addition, Section 4 shows how traditional moment selection criteria are altered when one moves away from a pure estimation setting to a setting with joint estimation and control. Section 5 presents a quantitative example. Section 6 considers a multivariate extension.

2. The Economic Setting

We consider a univariate parameter inference problem where the econometric model is exactly identified. The first subsection describes timing and technology assumptions. The second subsection illustrates how the general framework maps to a specific applied microeconometric problem.

2.1. Timing, Technology, and Beliefs

There is a real-world representative sample consisting of a continuum of atomistic agents ("firms") privately endowed with a policy-invariant ("deep") structural parameter. Knowledge of this parameter is sufficient for the government to set policy optimally.

An econometrician will observe an empirical moment derived from the measured actions of the sample firms. To fix ideas, one can think of the moment as being the sample mean of investment, new employees, R&D, or leverage. In practice, moments such as variance, skewness, or kurtosis may also be informative about deep firm-level parameters. In the context of indirect inference, the moment can be the coefficient obtained when firm decision variables are regressed on observable covariates, such as the coefficient on market-to-book (Q) in an investment regression. Examples are provided below.

The econometrician has developed a structural model and will match her model-implied moment with the observed empirical moment. Importantly, under conditions derived below, if the econometrician were to impose rational expectations in a fully internally consistent manner, this moment matching procedure would allow her to infer the true value of the deep parameter and the government would then be able to correctly determine the optimal policy.

The atomistic firms are rational, forward-looking, and act non-cooperatively. Each atomistic firm correctly understands it cannot change the moment observed by the econometrician by unilaterally changing its own action.

The deep parameter, denoted u, is common to all sample firms. However, this assumption does not preclude firm heterogeneity. For example, firms may be identical ex ante but face idiosyncratic shocks ex post. Alternatively, firms may face idiosyncratic shocks that alter their measured actions. Finally, firm-level parameters might be, say, multiples of a common aggregate parameter u, e.g. $u_i = \varepsilon_i u$ where ε_i is a firm-specific scalar known by firm i. An alternative technological assumption, not adopted here, is that each firm receives a noisy signal of the common parameter u. In such a setting, as in the present setting, parameter inference would need to account for feedback from inference to the policy variable.

The parameter u represents the realization of a random variable $\widetilde{u}$ with cumulative distribution function Ψ with a strictly positive density ψ on $\mathbb{R}$ with no atoms. The realized parameter u is privately observed by each of the sample firms, but unobservable to the econometrician and the government. Below, $\widehat{u}(u)$ denotes an equilibrium parameter inference by the econometrician in the event that $\widetilde{u} = u$, with $\widehat{u}(\cdot)$ denoting an equilibrium inference function.

Timing is as follows. When the model opens at time $t = 0$, the government policy variable is initially equal to the pre-determined *status-quo* $\gamma_0 \in \Gamma$ where the set of feasible government policies is $\Gamma \equiv (\underline{\gamma}, \overline{\gamma})$. Next, nature draws u according to the distribution function Ψ. Each sample firm i then chooses an optimal *pre-inference action* ϕ_i. This action can be multi-dimensional. The econometrician then observes the *empirical moment* m, which is derived from the pre-inference actions of the sample firms. Next, the econometrician will attempt to match her model-implied moment with the empirical moment, resulting in *parameter inference* $\widehat{u}$. The econometrician then reports $\widehat{u}$ to the government. All of these events take place at the initial time $t = 0$.

Time is either discrete or continuous and the horizon can be finite or infinite. There is an independent stochastic process d such that for all $t \geq 0$, $d_t \in \{0, 1\}$. Let

$$t^* \equiv \inf_{t \geq 0} \ d_t = 1.$$

At time t^*, the government enjoys a one-time opportunity to permanently re-set the policy variable, having already received the econometrician's report. At all prior dates, policy is fixed at the status quo γ_0. The *equilibrium discretionary policy* is denoted γ^*. Under the stated assumptions, government policy post-inference is a stochastic process $\widetilde{\gamma}_t$ with

$$\begin{aligned} t &< t^* \Rightarrow \widetilde{\gamma}_t = \gamma_0 \\ t &\geq t^* \Rightarrow \widetilde{\gamma}_t = \gamma^*. \end{aligned} \tag{1}$$

No sample firm receives any signal that is informative about γ^* aside from u. Thus, firm policy expectations are homogeneous. With this in mind, let γ denote the value of γ^* anticipated by the sample firms conditional upon their knowledge of u.

The optimal pre-inference action of firm i can be expressed as

$$\phi_i(u, \gamma; \gamma_0) \tag{2}$$

where the subscript i captures idiosyncratic shocks and the semi-colon separates variables from the constant γ_0.

It is assumed that observation of a continuum of sample firms is sufficient to ensure that any idiosyncratic shocks have no effect on the observed moment, so that m can be expressed as $m(u, \gamma; \gamma_0)$. For brevity, the constant γ_0 will be suppressed and the empirical moment will be represented by the following mapping:

$$m : \mathbb{R} \times \Gamma \to \mathbb{R}. \tag{3}$$

The first argument in the moment function m is the unknown parameter $u \in \mathbb{R}$. The second argument in the moment function is anticipated discretionary government policy $\gamma \in \Gamma$.

The following assumption ensures the setting considered is seemingly-ideal.

Assumption 1. *The model-implied moment function is identical to the empirical moment function $m : \mathbb{R} \times \Gamma \to \mathbb{R}$. Moreover, for each $\gamma \in \Gamma$, the function $m(\cdot, \gamma)$ is continuously differentiable and strictly monotonic.*

The first part of Assumption 1 states that the structural model is correct. In particular, from Assumption 1 it follows that if the model were to be parameterized with a correct stipulation of u and γ, the model-implied moment would match the empirical moment. The second part of Assumption 1 is the traditional structural identifying assumption that $m(\cdot, \gamma)$ is strictly monotonic.

We next characterize how the moment varies with anticipated discretionary government policy.

Assumption 2. *For each $u \in \mathbb{R}$, $m(u, \cdot)$ is a continuously differentiable strictly monotonic function.*

Notice, the setting considered is quite general. For example, as in Blume, Easley and O'Hara (1982), one can think of the sample firms as solving canonical finite or infinite horizon dynamic programming problems with differentiable policy functions where monotone comparative statics apply and carry over to m. Nevertheless, it is worth emphasizing that in order for Assumption 2 to hold, it must be the case that the sample firms are solving forward-looking problems in which anticipated discretionary government policy γ enters as a relevant parameter in their program, either through periodic payoff functions, constraint functions, and/or transition functions.

The function $g : \mathbb{R} \to \Gamma$ represents optimal discretionary government policy. If the government had the ability to directly observe u, its optimal discretionary policy would be $g(u)$. Of course, the sample firms will have already chosen their pre-inference actions ϕ_i. However, the government correctly understands that should it enjoy discretion, its policy choice γ^*, in addition to the parameter u, will determine the post-inference actions of the sample firms and/or other agents in the economy, e.g. future generations of firms. The function g represents the socially optimal u-contingent government policy in light of the relevant tradeoffs. The following assumption is imposed.

Assumption 3. *The optimal government policy g is a continuously differentiable strictly monotonic function mapping $\mathbb{R}$ onto Γ.*

The government is presumed to believe that the standard moment matching exercise will allow the econometrician to deliver a correct estimate of the unknown parameter. Critically, Assumption 1 would seem to imply that this confidence is justified. After all, the model moment function is equal to the empirical moment function, and the moment is monotone in the unknown parameter. We have the following assumption.

Assumption 4. *The government chooses discretionary policy optimally given its belief that for all $u \in \mathbb{R}$, $\widehat{u}(u) = u$.*

From Assumption 4 it follows that for all $u \in \mathbb{R}$, the *endogenous discretionary policy* of the government is

$$\gamma^*(u) = g[\widehat{u}(u)]. \tag{4}$$

An alternative interpretation of condition (4) is that the function g represents equilibrium policy outcomes from an extensive form game in which the econometrician's parameter estimate is fed into the political process. This alternative interpretation would not alter the characterization of bias below, but would necessarily rule out characterization of the welfare consequences of biased parameter inference.

We posit that the *real-world firms* form rational expectations. In particular, real-world firms know that the government may enjoy policy discretion at some future date. They also know the government will place full faith in the econometrician's structural parameter estimate $\widehat{u}$, and will then input this estimate into the policy function g. The following assumption formalizes this specification of firm beliefs.

Assumption 5 [Agent Rational Expectations]. *For all $u \in \mathbb{R}$, real-world firms correctly anticipate discretionary government policy, with*

$$\gamma(u) = \gamma^*(u) = g[\widehat{u}(u)]. \tag{5}$$

The first equality in the preceding equation ensures that $\gamma(\cdot)$ satisfies rational expectations. The second equality reflects how discretionary government policy γ^* will actually be formed in equilibrium, with $\widehat{u}(u)$ being fed into g. Effectively, under rational expectations, the real-world firms infer the econometrician's parameter estimate which allows them to correctly anticipate discretionary government policy.

From the preceding equation it follows that the empirical moment observed by the econometrician is:

$$m[u, \gamma(u)] = m[u, \gamma^*(u)] = m[u, g(\widehat{u}(u))]. \tag{6}$$

In reality, the post-inference government policy follows the stochastic process described in equation (1). The real-world sample firms have rational expectations and understand this. However, we assume the econometrician departs from rational expectations by parameterizing her structural model according to the status-quo. We have the following assumption.

Assumption 6 [Status Quo Parameterization]. *Firms inside the structural model anticipate that the status quo will be maintained even if the government enjoys policy discretion, with the belief $\gamma = \gamma_0$.*

Notice, by parameterizing her model according to the status quo, the econometrician implicitly treats the firms as being unaware of her own activities and the policy function they are intended to serve, informing the government's discretionary decisions. Below we analyze the implications for parameter inference and government policy.

From the preceding discussion it follows that for all $u \in \mathbb{R}$, the structural econometrician's parameter estimate will be derived from the following *inference equation*

$$m[u, \gamma^*(u)] = m[\widehat{u}(u), \gamma_0] \tag{7}$$

or

$$m[u, g(\widehat{u}(u))] = m[\widehat{u}(u), \gamma_0]. \tag{8}$$

The left side of the preceding equation is the real-world empirical moment. The empirical moment reflects the fact that the sample firms will choose their pre-inference actions optimally given the true parameter value u and their correct anticipation of discretionary government policy (Assumption 5). The right side of the preceding equation is the model-implied moment under the status quo parameterization (Assumption 6). The estimated parameter $\widehat{u}(u)$ is chosen so that the model implied moment is equal to the observed empirical moment.

Before proceeding, it is worthwhile to consider an alternative, more complex, motivation for the inference equation (8) since this equation serves as the foundation for all subsequent results regarding bias and bias correction. In particular, suppose instead the structural model does not treat government policy as fixed forever at γ_0 but instead treats government policy as an exogenous stochastic process as in, say, Keane and Wolpin (2002). To approximate such an inference approach within our framework, one can think of the structural model as treating government policy as an independent discrete-state Markov chain with one state, call it state 0, being the one real-world state in which the government will enjoy full policy discretion and follow the policy advice offered by the econometrician. The structural model then incorrectly treats government policy in state 0 as being an exogenous parameter γ_0 while the real-world firms understand that government policy in the discretionary state 0 will be endogenously set at $g(\widehat{u}(u))$. The inference equation (8) still applies in such a setting, and consequently, so do all the results that follow below. Having said this, it is clear that the approach of Keane and Wolpin (2002), while violating rational expectations in exercises of joint estimation and control, still offers an improvement over the common practice of treating policy as fixed forever at the status quo.

2.2. Example: Inferring Labor Adjustment Costs

At this stage it will be useful to fix ideas by considering a stripped-down example of the type of inference problem subsumed by our model. To this end, consider an econometrician who wants to estimate a labor adjustment cost parameter u based upon some empirical moment, say, the average change in firm or plant-level employment. This exercise is in the spirit of Hammermesh (1989), Blanchard and Portugal (2001), and Ejarque and Portugal (2007) who estimate parameters of labor adjustment cost functions and then use the estimates as the basis for making policy recommendations regarding labor market reforms. Although the focus of the example is on labor adjustment costs, similar arguments would apply to moment-based inference of capital stock adjustment cost parameters.

Let ϕ_i denote the number of workers hired by firm i. Firms face quadratic costs of bringing new employees onto their workforce, with the costs increasing in units of governmental regulation. The status quo features γ_0 units of regulation. The government will enjoy policy discretion with probability $p > 0$ and firms anticipate γ units of discretionary regulation. The sample firms make their hiring decisions before the policy uncertainty is resolved. Each "real-world" firm solves the following linear-quadratic program:

$$\max_{\phi_i} \quad \phi_i q - \frac{1}{2}[p\gamma + (1-p)\gamma_0]N(u)(\phi_i - \varepsilon_i)^2. \tag{9}$$

In the preceding equation, $q > 0$ represents the shadow value of an "installed" worker–the net present value of marginal product less wages. For simplicity, assume q is known to the econometrician.[9] The function N is, say, the normal cumulative distribution. This function is scaled by expected units of regulation. The term ε_i is mean-zero firm-specific shock. In this way, the structural estimation allows for heterogeneity.

Imposing rational expectations, with $\gamma = \gamma^*$, the econometrician observes the following empirical moment:

$$\int_i \phi_i di = m[u, \gamma^*(u)] = [pg(\widehat{u}(u)) + (1-p)\gamma_0]^{-1}[N(u)]^{-1}q. \tag{10}$$

[9] Or the econometrician is willing to rely upon existing estimates of this parameter.

The econometrician chooses her parameter estimate so that the model-implied moment is just equal to the observed empirical moment. The inference equation (8) is:

$$[pg(\widehat{u}(u)) + (1-p)\gamma_0]^{-1}[N(u)]^{-1}q = [\gamma_0]^{-1}[N(\widehat{u}(u))]^{-1}q. \tag{11}$$

Rearranging terms in the preceding equation we find

$$\widehat{u}(u) = N^{-1}\left[\frac{pg[\widehat{u}(u)] + (1-p)\gamma_0}{\gamma_0} \times N(u)\right]. \tag{12}$$

From the preceding equation it follows that

$$\widehat{u}(u) = u \Leftrightarrow g(u) = \gamma_0. \tag{13}$$

That is, parameter inference is unbiased at point u if and only if the status quo is actually optimal at that point. The next subsection offers a more general and precise characterization of bias.

3. Bias Characterization

This section characterizes the nature of parameter inference and associated policy outcomes if the structural model fails to impose the assumption that firms have rational expectations.

Before proceeding, it will be convenient to express the differential form of the inference equation. In particular, under technical conditions derived below, there will exist a continuously differentiable function $\widehat{u}(\cdot)$ satisfying the inference equation (8). Assuming such a function exists, we have the following differential form:

$$m_u[u, g(\widehat{u}(u))] + m_\gamma[u, g(\widehat{u}(u))]g'[\widehat{u}(u)]\widehat{u}'(u) = m_u[\widehat{u}(u), \gamma_0]\widehat{u}'(u). \tag{14}$$

The differential form of the inference equation makes clear the potential for bias. The right side captures the econometrician's faulty inference procedure which is predicated upon the incorrect assumption that firms expect the status quo to be maintained with probability 1. Thus, she incorrectly imputes any change in the observed moment to the direct effect as captured by the partial derivative, m_u. The left side of the preceding equation captures the true total differential of the empirical moment with respect to u. If u is perturbed, there will be a direct effect on the moment as captured by the first term, m_u. In addition, the empirical moment will vary due to the rational anticipation of firms that government policy will change based upon changes in the econometrician's parameter inference. This inference-policy feedback effect is captured by the second term on the left side of the equation ($m_\gamma g' \widehat{u}'$).

Let u_0 be the unique value of the parameter u at which a fully-informed government would find it optimal to implement the status quo policy γ_0. That is

$$u_0 \equiv g^{-1}(\gamma_0) \Leftrightarrow g(u_0) = \gamma_0. \tag{15}$$

Uniqueness of u_0 and invertibility follow from g being strictly monotone (Assumption 3).

The next proposition characterizes the realization(s) of the random variable $\widetilde{u}$ at which parameter inference will be unbiased.

Proposition 1. *Let the structural model be parameterized assuming government will implement* γ_0 *(the status quo) when it enjoys policy discretion. Parameter inference is unbiased at point* u *if and only if* $g(u) = \gamma_0$. *There is a unique point at which this occurs,* $u_0 \equiv g^{-1}(\gamma_0)$.

Proof. Referring to the inference equation (7), it follows from the strict monotonicity of m in its first argument that

$$\gamma^*(u) = \gamma_0 \Rightarrow \widehat{u}(u) = u.$$

Again referring to the inference equation (7), it follows from the strict monotonicity of m in its second argument that

$$\widehat{u}(u) = u \Rightarrow \gamma^*(u) = \gamma_0.$$

Finally if point u is a point such that parameter inference is unbiased and the status quo is optimal then it must be that

$$\gamma^*(u) = g(u) = \gamma_0.$$

From the strict monotonicity of g the unique point at which this occurs, u_0.$\blacksquare$

The intuition for the preceding result is as follows. At any realization of u other than u_0, real-world firms anticipate the government will implement a policy different from the status quo should it enjoy policy discretion. The real-world firms then change their optimal behavior accordingly, leading to changes in the observed moment. However, under Assumption 6, the econometrician fails to take the inference-policy feedback effect into account, leading to bias.

Having established parameter inference will only be unbiased at point u_0, the next proposition provides insight into the nature of bias at all other $u \in \mathbb{R}$.

Proposition 2. *Let the inference equation (7) be satisfied at point u by $\widehat{u}(u)$. If $m_u m_\gamma > 0$, then*

$$\begin{aligned} \gamma^*(u) &< \gamma_0 \Rightarrow \widehat{u}(u) < u \\ \gamma^*(u) &> \gamma_0 \Rightarrow \widehat{u}(u) > u. \end{aligned}$$

If $m_u m_\gamma < 0$, then

$$\begin{aligned} \gamma^*(u) &< \gamma_0 \Rightarrow \widehat{u}(u) > u \\ \gamma^*(u) &> \gamma_0 \Rightarrow \widehat{u}(u) < u. \end{aligned}$$

Proof. There are four cases to consider. Suppose first m is increasing in both arguments. Then from the inference equation (7) it follows

$$\begin{aligned} \gamma^*(u) &< \gamma_0 \Rightarrow m[u, \gamma_0] > m[u, \gamma^*(u)] \equiv m[\widehat{u}(u), \gamma_0] \Rightarrow \widehat{u}(u) < u \\ \gamma^*(u) &> \gamma_0 \Rightarrow m[u, \gamma_0] < m[u, \gamma^*(u)] \equiv m[\widehat{u}(u), \gamma_0] \Rightarrow \widehat{u}(u) > u. \end{aligned}$$

Suppose next m is decreasing in both arguments. Then

$$\begin{aligned} \gamma^*(u) &< \gamma_0 \Rightarrow m[u, \gamma_0] < m[u, \gamma^*(u)] \equiv m[\widehat{u}(u), \gamma_0] \Rightarrow \widehat{u}(u) < u \\ \gamma^*(u) &> \gamma_0 \Rightarrow m[u, \gamma_0] > m[u, \gamma^*(u)] \equiv m[\widehat{u}(u), \gamma_0] \Rightarrow \widehat{u}(u) > u. \end{aligned}$$

Suppose next m is decreasing in its first argument and increasing in its second argument. Then

$$\begin{aligned} \gamma^*(u) &< \gamma_0 \Rightarrow m[u, \gamma_0] > m[u, \gamma^*(u)] \equiv m[\widehat{u}(u), \gamma_0] \Rightarrow \widehat{u}(u) > u \\ \gamma^*(u) &> \gamma_0 \Rightarrow m[u, \gamma_0] < m[u, \gamma^*(u)] \equiv m[\widehat{u}(u), \gamma_0] \Rightarrow \widehat{u}(u) < u. \end{aligned}$$

Suppose finally m is increasing in its first argument and decreasing in its second argument. Then

$$\begin{aligned} \gamma^*(u) &< \gamma_0 \Rightarrow m[u, \gamma_0] < m[u, \gamma^*(u)] \equiv m[\widehat{u}(u), \gamma_0] \Rightarrow \widehat{u}(u) > u \\ \gamma^*(u) &> \gamma_0 \Rightarrow m[u, \gamma_0] > m[u, \gamma^*(u)] \equiv m[\widehat{u}(u), \gamma_0] \Rightarrow \widehat{u}(u) < u. \blacksquare \end{aligned}$$

The intuition behind the preceding result is as follows. Per Assumption 6, the econometrician's structural model incorrectly stipulates firm beliefs at any u at which the discretionary government policy will differ from the status quo. This incorrect stipulation of beliefs leads to incorrect inference. For example, taking the first part of the proposition, suppose the empirical moment function m is increasing (decreasing) in both arguments. Then if, say, $\gamma^*(u) > \gamma_0$, the moment will be higher (lower) than would be inferred based upon the direct effect m_u, causing $\widehat{u}$ to overshoot u. Taking the second part of the proposition, suppose $m_u > 0$ and $m_\gamma < 0$. Then if, say, $\gamma^*(u) > \gamma_0$, the moment will be lower than would be inferred based upon the direct effect m_u, causing $\widehat{u}$ to undershoot u.

The preceding proposition characterizes $\widehat{u}$ at a *particular point* u where the inference equation (7) has a solution. However, as shown below, the inference equation need not have a solution. With this in mind, the following lemma offers a sufficient condition for the existence of a (continuously differentiable) function $\widehat{u}(\cdot)$ satisfying the inference equation pointwise for all $u \in \mathbb{R}$.

Lemma 1. *Let $m_u m_\gamma < 0$ and $g' > 0$ or let $m_u m_\gamma > 0$ and $g' < 0$. Then there exists a continuously differentiable strictly monotonic increasing function $\widehat{u}(\cdot)$ satisfying the inference equation (7) for all $u \in \mathbb{R}$. The function $\widehat{u}(\cdot)$ has slope in $(0,1)$ at u_0.*

Proof. Consider the following function which is continuously differentiable in its two arguments

$$F(u, z) \equiv m[u, g(z)] - m(z, \gamma_0).$$

Any root z of the preceding equation represents a solution to the inference equation *(7)*. We know (Proposition 1) the root at u_0 is u_0. Consider next arbitrary $u \neq u_0$. Under the stated conditions it is readily verified that

$$\begin{aligned} F(u, u) &\equiv m[u, g(u)] - m(u, \gamma_0) \\ F(u, u_0) &\equiv m(u, \gamma_0) - m(u_0, \gamma_0) \end{aligned}$$

have opposite signs. From the Location of Roots Theorem, there exists a point $\widehat{u}$ solving the inference equation

$$F(u, \widehat{u}) = 0.$$

Moreover, under the stated conditions

$$\frac{\partial}{\partial \widehat{u}} F(u, \widehat{u}) = m_\gamma[u, g(\widehat{u})]g'(\widehat{u}) - m_u(\widehat{u}, \gamma_0) \neq 0.$$

It thus follows from the Implicit Function Theorem that there exists a continuously differentiable function $\widehat{u}(\cdot)$ defined on an interval I about the (arbitrary) point u such that

$$F[\widetilde{u}, \widehat{u}(\widetilde{u})] = 0 \quad \forall \quad \widetilde{u} \in I \tag{16}$$

and

$$\begin{aligned} \widehat{u}'(u) &= \frac{m_u[u, g(\widehat{u}(u))]}{m_u[\widehat{u}(u), \gamma_0] - m_\gamma[u, g(\widehat{u}(u))]g'[\widehat{u}(u)]} \\ &= \left[\frac{m_u[\widehat{u}(u), \gamma_0]}{m_u[u, g(\widehat{u}(u))]} - \frac{m_\gamma[u, g(\widehat{u}(u))]g'[\widehat{u}(u)]}{m_u[u, g(\widehat{u}(u))]}\right]^{-1}. \end{aligned} \tag{17}$$

Notice, under the stated conditions, the term in square brackets in the preceding equation is strictly positive, implying the derivative of the function $\widehat{u}$ is positive. Finally, the last statement in the lemma follows from

$$\begin{aligned} \widehat{u}'(u_0) &= \frac{m_u[u_0, g(\widehat{u}(u_0))]}{m_u[\widehat{u}(u_0), \gamma_0] - m_\gamma[u_0, g(\widehat{u}(u_0))]g'[\widehat{u}(u_0)]} \\ &= \frac{m_u[u_0, g(u_0)]}{m_u(u_0, \gamma_0) - m_\gamma[u_0, g(u_0)]g'(u_0)} \\ &= \left[1 - \frac{m_\gamma[u_0, g(u_0)]g'(u_0)}{m_u(u_0, \gamma_0)}\right]^{-1}. \blacksquare \end{aligned} \tag{18}$$

To illustrate the preceding lemma, and many that follow, it will be useful to define a *linear technology:*

$$\begin{aligned} m(u, \gamma) &\equiv \alpha u + \beta\gamma \\ g(\widehat{u}) &\equiv \kappa\widehat{u} \end{aligned} \tag{19}$$

where α, β and κ are arbitrary nonzero constants. Under the linear technology, the inference equation (8) can be written as

$$u + \kappa\widehat{u}(u) = \widehat{u}(u) + \gamma_0.$$

From equation (15) it follows that here $\gamma_0 = \kappa u_0$. Using this fact, and rearranging terms in the preceding equation, the inference equation can be expressed as

$$\alpha u - \beta\kappa u_0 = (\alpha - \beta\kappa)\widehat{u}(u). \tag{20}$$

If $\alpha = \beta\kappa$, the preceding equation does not have a solution at any point other than u_0. Under the conditions in Lemma 1, $\alpha \neq \beta\kappa$. In fact, under the conditions specified in the lemma, α and $\beta\kappa$ have different signs. With $\alpha \neq \beta\kappa$, the solution to the linear technology inference equation is

$$\widehat{u}(u) = \frac{\alpha u - \beta\kappa u_0}{\alpha - \beta\kappa} = u + \frac{\beta\kappa(u - u_0)}{\alpha - \beta\kappa}. \tag{21}$$

Under the conditions stated in Lemma 1, $\widehat{u}'$ is some constant in $(0, 1)$.

Lemma 1 leads directly to the following proposition.

Proposition 3. *Let $m_u m_\gamma < 0$ and $g' > 0$ or let $m_u m_\gamma > 0$ and $g' < 0$. Then there exists a continuously differentiable strictly monotonic increasing function $\widehat{u}(\cdot)$ satisfying the inference equation. For all $u < u_0$, $\widehat{u}(u) \in (u, u_0)$ and for all $u > u_0$, $\widehat{u}(u) \in (u_0, u)$. If g is increasing, then $u < u_0$ implies $\gamma^*(u) \in (g(u), \gamma_0)$ and $u > u_0$ implies $\gamma^*(u) \in (\gamma_0, g(u))$. If g is decreasing, then $u < u_0$ implies $\gamma^*(u) \in (\gamma_0, g(u))$ and $u > u_0$ implies $\gamma^*(u) \in (g(u), \gamma_0)$.*

Proof. The first statement in the proposition is from Lemma 1. Next note that $\widehat{u}'(u_0) \in (0, 1)$. It follows that for u on the left neighborhood of u_0, $\widehat{u}(u) \in (u, u_0)$ and for u on the right neighborhood of u_0, $\widehat{u}(u) \in (u_0, u)$. From the continuity of $\widehat{u}(\cdot)$ and Proposition 1 it follows that for all $u < u_0$, $\widehat{u}(u) > u$ and for all $u > u_0$, $\widehat{u}(u) < u$. From the strict monotonicity of $\widehat{u}(\cdot)$ it follows that for all $u < u_0$, $\widehat{u}(u) < u_0$ and for all $u > u_0$, $\widehat{u}(u) > u_0$. The final two statements in the proposition follow from the fact that $\gamma^* = g(\widehat{u})$.■

Inspection of equation (14) reveals the intuition for the preceding proposition. Under the stated assumptions, the second term on the left side of the differential form of the inference equation (14) dampens the sensitivity of the moment to changes in u–an effect ignored by the econometrician. She will then incorrectly impute the small changes in the moment to small changes in u. That is, $\widehat{u}$ will tend to have a slope less than unity, with $\widehat{u}$ overshooting for $u < u_0$ and undershooting for $u > u_0$.

These effects are illustrated in Figures 1, 2 and 3 which consider the linear technology with $m = u - \gamma$ and $g = \widehat{u}/2$, with $u_0 = 0$. Equation (21) pins down the inference function here, with $\widehat{u}'(u) = 2/3$. Figure 1 contrasts the true empirical moment function $m[u, g(\widehat{u}(u))]$ and the econometrician's model-implied moment function $m(u, \gamma_0)$. The former accounts for policy feedback and the latter fails to do so. Here the econometrician incorrectly imputes the dampened sensitivity of the observed moment to changes in u to small changes in u. Figure 2 shows the resulting single crossing of $\widehat{u}$ with the 45 degree line from above, consistent with the notion of dampened sensitivity. Finally, since g has here been assumed to be increasing, Figure 3 shows the resulting policy overshooting relative to the optimal policy for low values of u and undershooting relative to the optimal policy for high values of u.

We next consider the nature of inference and policy bias under alternative technologies. However, before doing so, we must establish a sufficient condition for the existence of a well-behaved solution to the inference equation. After all, if we consider departures from the technologies assumed in the preceding proposition, it is possible that there is no solution to the inference equation. To see this, consider the linear technology and suppose that, departing from the preceding two propositions, α and β have the same sign and $\kappa > 0$ or α and β have different signs and $\kappa > 0$. In either case, it is possible that $\alpha = \beta\kappa$ so that there is no solution to the inference equation. With such a possibility in mind, the next lemma provides a sufficient condition for the existence of a continuously-differentiable solution to the inference equation.

Lemma 2. *If*

$$m_1(x, \gamma_0) \neq m_2[u, g(x)]g'(x) \quad \forall\ (x, u) \in \mathbb{R} \times \mathbb{R}, \tag{22}$$

then there exists a continuously differentiable strictly monotone function $\widehat{u}(\cdot)$ *satisfying the inference equation (7) for all* $u \in \mathbb{R}$.

Proof. Define the following candidate solution to the inference equation

$$\widehat{u}(u) \equiv u_0 + \int_{u_0}^{u} \frac{m_u[\upsilon, g(\widehat{u}(\upsilon))]}{m_u[\widehat{u}(\upsilon), \gamma_0] - m_\gamma[\upsilon, g(\widehat{u}(\upsilon))]g'[\widehat{u}(\upsilon)]} d\upsilon.$$

Since here $\widehat{u}(u_0) = u_0$, the candidate solution satisfies the inference equation at u_0 (Proposition 1). Further, under the stated assumptions, the candidate solution has a well-defined derivative at all points, given in equation (17). Rearranging terms in equation (17), it follows that the candidate solution satisfies the differential form of the inference equation (14) point-wise. Thus, $\widehat{u}$ is a continuous and differentiable solution to the inference equation. Moreover, $\widehat{u}$ is continuously differentiable since m and g are continuously differentiable. Finally, the sign of the numerator in equation (17) is constant. And the sign of the denominator of this same equation cannot change since, by the Location of Roots Theorem, this would imply the existence of an intermediate point such that the inequality in equation (22) is violated. Thus, $\widehat{u}$ must be strictly monotonic.■

To take a specific example, if the conditions of Lemma 2 were to be satisfied in the context of the linear technology (equation (19)), then it follows $\alpha \neq \beta\kappa$ and the linear technology inference function (21) along with its derivative would be well-defined.

We have the following proposition.

Proposition 4. *Let $m_u m_\gamma > 0$ and $g' > 0$ or let $m_u m_\gamma < 0$ and $g' < 0$, with condition (22) being satisfied. If*

$$\frac{m_\gamma(u_0, \gamma_0) g'(u_0)}{m_u(u_0, \gamma_0)} < 1,$$

there exists a continuously differentiable strictly monotonic increasing function $\widehat{u}(\cdot)$ satisfying the inference equation. For all $u < u_0$, $\widehat{u}(u) < u$ and for all $u > u_0$, $\widehat{u}(u) > u$. If g is increasing then $u < u_0$ implies $\gamma^*(u) < g(u)$ and $u > u_0$ implies $\gamma^*(u) > g(u)$. If g is decreasing then $u < u_0$ implies $\gamma^*(u) > g(u)$ and $u > u_0$ implies $\gamma^*(u) < g(u)$.

If

$$\frac{m_\gamma(u_0, \gamma_0) g'(u_0)}{m_u(u_0, \gamma_0)} > 1,$$

then there exists a continuously differentiable strictly monotonic decreasing function $\widehat{u}(\cdot)$ satisfying the inference equation. For all $u < u_0$, $\widehat{u}(u) > u_0 > u$ and for all $u > u_0$, $\widehat{u}(u) < u_0 < u$. If g is increasing then $u < u_0$ implies $\gamma^*(u) > \gamma_0 > g(u)$ and $u > u_0$ implies $\gamma^*(u) < \gamma_0 < g(u)$. If g is decreasing then $u < u_0$ implies $\gamma^*(u) < \gamma_0 < g(u)$ and $u > u_0$ implies $\gamma^*(u) > \gamma_0 > g(u)$.

Proof. From Lemma 2 there exists a continuously differentiable strictly monotonic solution to the inference equation. From the final line in equation (18) it follows

$$\frac{m_\gamma(u_0, \gamma_0) g'(u_0)}{m_u(u_0, \gamma_0)} < 1 \Rightarrow \widehat{u}'(u_0) > 1.$$

Considering this case, $\widehat{u}$ must be strictly monotone increasing. Moreover, on the left neighborhood of u_0, $\widehat{u}(u) < u$ and on the right neighborhood of u_0, $\widehat{u}(u) > 0$. From the continuity of $\widehat{u}$ and Proposition 1 it follows that for all $u < u_0$, $\widehat{u}(u) < u$ and for all $u > u_0$, $\widehat{u}(u) > u$.

For the second part of the proposition, note that

$$\frac{m_\gamma(u_0, \gamma_0) g'(u_0)}{m_u(u_0, \gamma_0)} > 1 \Rightarrow \widehat{u}'(u_0) < 0.$$

Considering this case, $\widehat{u}$ must be strictly monotone decreasing. It follows that for all $u < u_0$, $\widehat{u}(u) > u_0 > u$ and for all $u > u_0$, $\widehat{u}(u) < u_0 < u$. The clauses pertaining to discretionary government policy follow from the fact that $\gamma^* = g(\widehat{u})$.■

Inspection of equation (14) reveals the intuition for the first part of the preceding proposition. Under the posited technologies, the policy feedback effect causes the observed moment to be more sensitive to changes in u than is understood by the econometrician. She will then incorrectly impute large changes in the moment to large changes in u. That is, $\widehat{u}$ will tend to have a slope in excess of unity, so that $\widehat{u}$ undershoots for $u < u_0$ and overshoots for $u > u_0$. In other words, the function $\widehat{u}(u)$ will cross the function u at the point u_0 from below.

These effects are illustrated in Figures 4, 5 and 6 which consider the linear technology with $m = u + \gamma$ and $g = \widehat{u}/2$, with $u_0 = 0$. Equation (21) pins down the inference function here, with $\widehat{u}'(u) = 2$. Figure 4 shows how the econometrician will incorrectly impute large changes in the moment to large changes in u. Figure 5 shows the resulting single crossing of $\widehat{u}$ with u from below. Finally, since g has here been assumed to be increasing, Figure 6 shows the resulting policy undershooting for low values of u and overshooting for high values of u.

The second part of the preceding proposition is illustrated most vividly by considering a particular example. To this end, consider the same linear moment $m = u + \gamma$ but now assume $g = 2\widehat{u}$, with $u_0 = 0$. That is, in the case being considered, discretionary government policy is more sensitive to the inferred value of the structural parameter. Equation (21) pins down the inference function here, with $\widehat{u}(u) = -u$ for all u. Notice, here we have a situation where the inferred value of the parameter has the wrong sign with probability 1. Of course, this implies that discretionary government policy will move in exactly the opposite direction relative to what is optimal.

Figures 7, 8, and 9 depict the nature of inference under this technology. For example, suppose the realized value is $u = 5$. Firms conjecture the econometrician will infer $\widehat{u}(5) = -5$ and anticipate discretionary governmental policy will be $\gamma = 2\widehat{u} = -10$. The observed moment will be $m = u + \gamma = 5 - 10 = -5$. The econometrician incorrectly believes she is observing $m = u + 2u_0 = u$ and so indeed draws the inference conjectured by the firms, with $\widehat{u} = -5$. The government then implements $\gamma^* = -10$, consistent with the policy anticipated by the real-world firms.

4. Joint Estimation and Control under Rational Expectations

This section considers whether and how the econometrician can achieve unbiased parameter inference.

4.1. Avoiding Bias and Achieving Optimality

A natural to ask is whether it is possible to achieve unbiased parameter inference in the setting considered. Introspection suggests a ready solution. The underlying source of biased parameter inference in the preceding section was the failure of the econometrician to parameterize her model in a manner consistent with the rational expectations held by the firms (Assumption 6). Therefore, achieving unbiased inference would seem to necessitate "parameterizing" expectations correctly– with the issue being that the policy expectation is correctly understood as a function, rather than a parameter. Indeed, we have the following lemma.

Lemma 3. *If firms anticipate monotone discretionary policy outcomes $\gamma^{**}(\cdot)$, then parameter inference will be unbiased for all $u \in \mathbb{R}$ only if the structural model specifies discretionary policy outcomes as $\gamma^{**}(\cdot)$, with resulting rational expectations inference equation*

$$m[u, \gamma^{**}(u)] = m[\widehat{u}(u), \gamma^{**}(u)]. \tag{23}$$

Proof. Suppose the structural model specifies firm beliefs according to some function $\widetilde{\gamma}(\cdot)$. Then the inference equation will be

$$m[u, \gamma^{**}(u)] = m[\widehat{u}(u), \widetilde{\gamma}(\widehat{u}(u))]. \tag{24}$$

Thus

$$\widehat{u}(u) = u \Rightarrow m[u, \gamma^{**}(u)] = m[u, \widetilde{\gamma}(u)] \Rightarrow \widetilde{\gamma} = \gamma^{**}. \tag{25}$$

The second implication follows from the strict monotonicity of m in its second argument.■

Of course, the government's ultimate objective is not to achieve unbiased parameter inference but rather to implement the optimal policy when it enjoys discretion. Therefore, the government would like to construct a rational expectations equilibrium predicated upon correct inference *and* firms anticipating a specific endogenous outcome

$$\gamma^{**}(\cdot) = g(\cdot).$$

But a necessary condition for correct parameter inference to be feasible for all u is that the empirical moment be invertible. To this end, let

$$\mu(u) \equiv m[u, g(u)]. \tag{26}$$

We then have the following proposition.

Proposition 5. *Let the empirical moment $\mu(\cdot)$ (equation (26))be strictly monotone. Then parameter inference will be unbiased for all $u \in \mathbb{R}$ if and only if the structural model specifies discretionary policy outcomes as $g(\cdot)$.*

Proof. The "only if" part of the proposition follows from Lemma 3. For sufficiency, suppose the structural model specifies firm beliefs according to some function $\widetilde{\gamma}(\cdot)$. Then the inference equation will be

$$m[u, g(u)] = m[\widehat{u}(u), \widetilde{\gamma}(\widehat{u}(u))]. \tag{27}$$

For sufficiency, note

$$\widetilde{\gamma} = g \Rightarrow m[u, g(u)] = m[\widehat{u}(u), g(\widehat{u}(u))] \Rightarrow \widehat{u}(u) = u. \blacksquare$$

It follows that in order for the econometrician to avoid bias and achieve first-best, she must replace the faulty inference equation (7) with the rational expectations inference equation

$$m[u, g(u)] = m[\widehat{u}(u), g(u)]. \tag{28}$$

Of course, the measured agents must understand the econometrician's procedure. Formally, in a rational expectations equilibrium there is no need for any agent to make a speech. Nevertheless, heuristically, in support of the postulated equilibrium, the econometrician could be understood as making the following speech to the firms.

> I the structural econometrician will correctly infer the true value of the parameter u from the observation of the moment m that your actions generate. Further, armed with my correct inference, the government will implement the optimal policy $g(u)$ should it enjoy policy discretion. And now that I have made this speech to you, I know that you know I will do this, and so you should anticipate $g(u)$ as the discretionary government policy and, thus, act accordingly.

To further aid intuition, it is useful to express the rational expectations inference equation (28) in differential form:

$$m_u[u, g(u)] + m_\gamma[u, g(u)]g'(u) = m_u[\widehat{u}(u), g(u)]\widehat{u}'(u) + m_\gamma[u, g(u)]g'(u). \tag{29}$$

The left side of the preceding equation reflects how the moment actually changes with u, and the right side reflects how the structural model treats the moment as changing with u. The econometrician's structural model of firm behavior now takes into account firm expectations regarding policy recommendations, while the "counterfactuals" approach failed to do so.

4.2. Gallant and Tauchen Revisited

In the title to their important paper, Gallant and Tauchen (1996) pose a question often asked by structural modellers: "Which Moment to Match?" An overarching message of our paper is that the nature of econometric inference changes fundamentally if one is attempting joint estimation and control, rather than simply attempting estimation. This message carries over to moment selection.

To illustrate, consider an econometrician operating in a world with linear technologies, with two competing moments being considered candidates for matching. In particular, suppose the optimal government policy is κu, where moments 1 and 2 have the following forms, respectively:

$$\begin{aligned} m_1 &\equiv \beta_1 \gamma \\ m_2 &\equiv \alpha_2 u + \beta_2 \gamma \\ \alpha_2 &\equiv -\beta_2 \kappa. \end{aligned}$$

According to the traditional moment selection criteria, moment 1 would be discarded since it violates the standard moment monotonicity condition (Assumption 1). In particular, according to the traditional moment selection criteria, moment 1 would be viewed as completely uninformative about the unknown parameter. In contrast, moment 2 would be viewed as informative about the unknown parameter.

But recall, the econometrician is engaged in an exercise of joint estimation and control, with the government attempting to achieve first-best. In this context, moment 1 is informative and moment 2 is uninformative. In particular, consider a conjectured rational expectations equilibrium with correct inference and first-best policy implementation. In such an equilibrium the two moments can be expressed as univariate functions of the unknown parameter. We have

$$\begin{aligned} \mu_1 &= \beta_1 \gamma^{**}(u) = \beta_1 g(u) = \beta_1 \kappa u \qquad (30) \\ \mu_2 &= \alpha_2 u + \beta_2 \gamma^{**}(u) = \alpha_2 u + \beta_2 g(u) = [\alpha_2 + \beta_2 \kappa] u = 0. \end{aligned}$$

Notice, we have here a situation where without policy feedback, moment 2 is informative and moment 1 is uninformative. Conversely, with policy feedback, moment 2 is uninformative and moment 1 is informative. Strikingly, moment 2 can be highly informative about the true value of the unknown parameter solely due to its sensitivity to the governmental policy variable. Intuitively, as u changes, so too does governmental policy in equilibrium, and this causes firm behavior to change in a manner informative about u.

We thus have the following proposition.

Proposition 6. *Monotonicity of the moment function $m(\cdot, \gamma)$ is neither necessary nor sufficient for m to be informative about the unknown parameter with joint estimation of u and control of γ^*.*

4.3. An Algorithmic Approach to Structural Inference

This section considers a small departure from rational expectations' communism of models. In particular, consider the same information structure as above, but assume that, unlike the other agents, the econometrician does not know the government policy function g. Rather, the econometrician will report her inference $\widehat{u}$ to the government. The government will then "tentatively adopt" a policy $g(\widehat{u})$. Based on this, the econometrician can reconsider her inference.

Intuitively, the econometrician may not have full knowledge of the government's objectives. However, she should be able to recognize the inconsistency that is the source of biased inference (Section 3): The structural model is predicated upon the assumption that the status quo will be implemented even when the government enjoys policy discretion, and yet the government prefers the policy $g(\widehat{u}) \neq \gamma_0$ based on the inferred value of the parameter.

The question we now address is whether, despite not knowing g, the econometrician can iterate to (approximately) correct inference of u, so that the firms will be justified in conjecturing a rational expectations equilibrium in which policy converges to first-best, with $\gamma^*(u)$ arbitrarily close to $g(u)$. To this end, we propose the following *Algorithmic Inference Approach:*

- Start iteration $n \in \{1, 2, 3, ...\}$ with parameterized discretionary policy γ_n;
- Draw inference $\widehat{u}_n$ solving

$$m_{observed} = m(\widehat{u}_n, \gamma_n); \tag{31}$$

- Observe the tentative government policy $g(\widehat{u}_n)$ and define this to be γ_{n+1};

- Iterate until (approximate) internal consistency, $\left|\gamma_{n+1} - \gamma_n\right| < \epsilon$ for ϵ arbitrarily small.

We have the following proposition showing that if μ (equation (26)) is strictly monotonic, elimination of internal inconsistency is sufficient to ensure correct inference and optimal government policy.

Proposition 7. *Let μ (equation (26)) be strictly monotonic. At the n-th iteration, let the structural model be parameterized assuming government will implement γ_n should it enjoy policy discretion. The resulting inference $\widehat{u}_n$ will be equal to the true parameter u if and only if $\widehat{u}_n$ rationalizes γ_n so that policy convergence obtains with $\gamma_n = g(\widehat{u}_n) \equiv \gamma_{n+1}$.*

Proof. To establish sufficiency suppose $\gamma_n = g(\widehat{u}_n)$. Under the stated conditions, the inference equation (7) can be rewritten as

$$m_{observed} = m[\widehat{u}_n, g(\widehat{u}_n)] \Rightarrow m_{observed} = \mu(\widehat{u}_n).$$

From monotonicity of μ, the unique value at which the observed moment matches the model-implied moment is the true u. To establish necessity, suppose $\gamma_n \neq g(\widehat{u}_n)$. It then follows from the moment matching equation and monotonicity of m in its second argument that

$$m_{observed} = m(\widehat{u}_n, \gamma_n) \neq m[\widehat{u}_n, g(\widehat{u}_n)] \equiv \mu(\widehat{u}_n).$$

Since $m_{observed} \neq \mu(\widehat{u}_n)$ it follows $\widehat{u}_n \neq u$.■

Of course, in practice, iteration will generally continue until approximate convergence. Therefore, it is interesting to evaluate the convergence properties of the preceding algorithm. Rather than do so numerically with arbitrary examples, we first consider below iterating on the preceding algorithm in the case of the linear technology. To begin, note that iterating on γ_n values is equivalent to iterating on the u values that would justify them, e.g. $\kappa u_{n+1} \equiv \gamma_{n+1}$. Thus, from the statement of the algorithm:

$$\kappa u_{n+1} \equiv \gamma_{n+1} = \kappa \widehat{u}_n \Rightarrow u_{n+1} = \widehat{u}_n.$$

In the posited rational expectations equilibrium, with first-best policy conjectured by the firms, the inference equation at iteration $n+1$ is

$$m[u, g(u)] = m[\widehat{u}_{n+1}, \gamma_{n+1}]. \tag{32}$$

With the linear technology, the preceding equation can be expressed as follows

$$\alpha u + \beta\kappa u = \alpha\widehat{u}_{n+1} + \beta\kappa\widehat{u}_n. \tag{33}$$

Iterating on the preceding equation we have the following lemma which shows that the proposed algorithm will converge to the truth provided the policy feedback effect is sufficiently weak relative to the direct effect.

Lemma 4. *Under the linear technology (equation (19)), the Algorithmic Inference Approach yields inference at the n-th iteration equal to*

$$\widehat{u}_n = u + \left(-\frac{\beta\kappa}{\alpha}\right)^n (u_1 - u). \tag{34}$$

The algorithm converges to the true parameter u for all $u \in \mathbb{R}$ for all starting points $u_1 \in \mathbb{R}$ if and only if

$$\left|\frac{\beta\kappa}{\alpha}\right| < 1.$$

In fact, Lemma 4 is a special case of a more general convergence condition which relies on bounding the policy feedback effect, as we show next.

Proposition 8. *The Algorithmic Inference converges to the true parameter u for all $u \in \mathbb{R}$ for all starting points $\gamma_1 \in \Gamma$ if*

$$\left|\frac{m_\gamma g'}{m_u}\right| < 1.$$

Proof. The inference equation is

$$m[u, g(u)] - m[\widehat{u}_n, \gamma_n] = 0.$$

The preceding equation can be rewritten as

$$\{m[u, g(u)] - m[\widehat{u}_n, g(u)]\} + \{m[\widehat{u}_n, g(u)] - m[\widehat{u}_n, \gamma_n]\} = 0.$$

From the mean value theorem, for each iteration n, there exists x_n between $\widehat{u}_n$ and u, and there exists g_n between $g(u)$ and γ_n such that

$$m_u[x_n, g(u)](u - \widehat{u}_n) + m_\gamma(\widehat{u}_n, g_n)[g(u) - g(\widehat{u}_{n-1})] = 0$$

Applying the mean value theorem to the final term in the preceding equation, we know that for each iteration n there exists $z_n \in$between u and $\widehat{u}_{n-1}$ such that

$$m_u \left[x_n, g\left(u\right)\right] \left(u - \widehat{u}_n\right) + m_\gamma \left(\widehat{u}_n, g_n\right) g'(z_n) \left(u - \widehat{u}_{n-1}\right) = 0.$$

Rearranging terms in the preceding equation, we find that at each iteration n

$$u - \widehat{u}_n = -\frac{m_\gamma \left[\widehat{u}_n, g_n\right] g'(z_n)}{m_u \left[x_n, g\left(u\right)\right]} \left(u - \widehat{u}_{n-1}\right).$$

Under the stated condition $\widehat{u}_n$ converges to u.■

5. Quantitative Example

This section considers an econometrician seeking to estimate unobserved costs of corporate bankruptcy based upon the financial policies adopted by corporations. Understanding the magnitude of bankruptcy costs is important for a number of reasons. First, to the extent that bankruptcy costs are deadweight losses, rather than transfers, their magnitude is directly relevant for assessing the efficiency costs of corporate leverage, as well as tax-induced leverage increases. For example, in making the case for the Bush Administration Treasury for integration of the individual and corporate tax systems, Hubbard (1993) contended, "tax-induced distortions in corporations' comparisons of nontax advantages and disadvantages of debt entail significant efficiency costs." Second, the magnitude of bankruptcy costs is indirectly relevant to the tax authority estimating revenues. After all, higher bankruptcy costs serve as a counterweight to tax benefits of debt, discouraging firms from taking on extremely high leverage. For example, Gruber and Rauh (2007) estimate the tax elasticity of corporate income is only -0.2, evidence that would appear to contradict Hubbard's notion that corporations aggressively change capital structures in response to tax incentives.

Early models, such as that of Stiglitz (1973), failed to deliver interior optimal leverage ratios. Lacking interior optimal leverage ratios, computational general equilibrium (CGE) models, e.g. Ballard, et. al (1985), posited exogenous financing rules. In the absence of closed models, public finance economists such as Gordon and MacKie-Mason (1990) and Nadeau (1993) were forced into positing ad hoc costs of financial distress. In an important contribution, Leland (1994) showed how to develop a tractable logically closed model of capital structure for firms facing taxation and costs of distress using contingent-claims pricing methods.

In this section, we use Leland's canonical framework to illustrate the magnitude of bias that can arise if the structural modeler fails to impose rational expectations. To this end, consider a government that is interested in setting the corporate income tax in a way that is optimal according to its objective function. The magnitude of financial distress costs is clearly relevant here since, as argued above, the magnitude of these costs determines efficiency costs of corporate leverage, as well as having a bearing on the present value of corporate income tax collections.

With this economic setting in mind, consider a structural econometrician who will observe the financing policies adopted by a set of homogeneous firms funding new investments during the pre-inference stage.[10] Specifically, the econometrician will measure the mean interest coverage ratio, as measured by the ratio of EBIT to interest expense. As shown below, this moment is directly informative about bankruptcy costs.

Consider first the decision problem of the firms. Each firm will choose a promised instantaneous coupon on a consol bond, denoted ϕ. The firm will use the debt proceeds plus equity injections to fund a new investment, as is standard in project finance settings. We assume parameters are such that the investment has positive net present value. Formally, the new investment has positive net present value if the value of the levered enterprise exceeds the cost of the investment.

Debt enjoys a tax advantage, with interest being a deductible expense on the corporate income tax return. Consequently, each instant it is alive, the project firm will capture a gross tax shield equal to $\phi\widetilde{\gamma}$, with the variable $\widetilde{\gamma}$ representing the corporate income tax rate that will be implemented just after the econometrician completes her parameter inference. The firm must weigh this debt tax shield benefit against costs of financial distress. In particular, in the event of EBIT being insufficient to service the coupon, the firm's debt will be cancelled and bondholders will recover the unlevered firm value net of deadweight bankruptcy costs representing a fraction $N(u)$ of unlevered firm value. The function N here is the standard normal cumulative distribution function.

Suppose firm EBIT follows a geometric Brownian motion with drift μ, volatility σ, and initial value normalized at 1. The risk-free rate is denoted r. The objective is to maximize levered project value. Or equivalently, firms maximize expected tax shield value minus expected default costs.

[10] The optimal coupon is linear in EBIT so coverage ratios will be equal if EBIT levels differ.

Letting γ represent the anticipated tax rate, firms solve the following program

$$\max_{\phi} \quad \gamma\phi\left(\frac{1}{r}\right)(1-\phi^{-\lambda}) - N(u)\frac{\phi(1-\gamma)}{r-\mu}\phi^{-\lambda}. \tag{35}$$

where λ is the negative root of the following quadratic equation

$$\frac{1}{2}\sigma^2\lambda^2 + \left(\mu - \frac{1}{2}\sigma^2\right)\lambda - r = 0. \tag{36}$$

Note, the first term in the objective function captures tax shield value and the second term captures bankruptcy costs. Effectively, the tax shield represents an annuity that expires at the first passage of EBIT to the coupon from above. At this same point in time, bankruptcy costs incurred. This explains the presence of the term $\phi^{-\lambda}$ in the objective function, which measures the price at date zero of a so-called primitive claim paying 1 the first passage of EBIT to the coupon from above.

The first-order condition for the optimal coupon entails equating marginal tax benefits with marginal bankruptcy costs. In particular, the optimal coupon satisfies

$$\left(\frac{\gamma}{r}\right)[1-(1-\lambda)\phi^{-\lambda}] = (1-\lambda)N(u)\frac{(1-\gamma)}{r-\mu}\phi^{-\lambda}. \tag{37}$$

Rearranging terms in the preceding equation, it follows the optimal coupon is

$$\phi^* = (1-\lambda)^{1/\lambda}\left[1 + N(u)\frac{(1-\gamma)}{r-\mu}\frac{r}{\gamma}\right]^{1/\lambda}. \tag{38}$$

The moment observed by the econometrician, the mean interest coverage ratio, is $1/\phi^*$. Thus, in the present setting

$$m(u,\gamma) \equiv \mathbb{E}[\phi^{-1}] = (1-\lambda)^{-1/\lambda}\left[1 + N(u)\frac{(1-\gamma)}{r-\mu}\frac{r}{\gamma}\right]^{-1/\lambda}. \tag{39}$$

Notice, in this particular case, $m_u(u,\gamma) > 0$ and $m_\gamma(u,\gamma) < 0$. That is, the optimal interest coverage ratio is increasing in bankruptcy costs and decreasing in the tax rate.

Suppose now that the structural econometrician, who recommended the Trump tax cut, failed to impose the assumption that firms have rational expectations ($\gamma = \gamma^*$). Specifically, suppose the econometrician treated the tax change as a counterfactual event and parameterized the model using the status quo tax rate. In the present context, the inference equation (7) takes the form

$$\begin{aligned} m[u,\gamma^*(u)] &= (1-\lambda)^{-1/\lambda}\left[1 + N(u)\frac{[1-\gamma^*(u)]}{r-\mu}\frac{r}{\gamma^*(u)}\right]^{-1/\lambda} \\ &= (1-\lambda)^{-1/\lambda}\left[1 + N(\widehat{u})\frac{(1-\gamma_0)}{r-\mu}\frac{r}{\gamma_0}\right]^{-1/\lambda} = m(\widehat{u},\gamma_0). \end{aligned} \tag{40}$$

Cancelling terms in the preceding equation and solving one obtains

$$N(\widehat{u}) = \frac{[1-\gamma^*(u)]/\gamma^*(u)}{(1-\gamma_0)/\gamma_0} \times N(u). \tag{41}$$

How important quantitatively is the bias implied by the preceding equation? Following Goldstein, Ju and Leland (2001) we can approximate the effect of personal taxes by setting γ_0 and γ^* based upon the Miller (1977) debt tax shield formula. In particular, let γ_c denote the corporate tax rate, γ_e denote the equityholder tax rate, and γ_d denote the debtholder tax rate. The Miller debt tax shield value is

$$\gamma = 1 - \frac{(1-\gamma_c)(1-\gamma_e)}{(1-\gamma_d)}. \tag{42}$$

Goldstein, Ju and Leland (2001) estimate $\gamma_c = 35\%$, $\gamma_e = 20\%$ and $\gamma_d = 35\%$. These parameter values are reflective of the status quo before the Trump corporate tax cut, which implies the status quo policy value is $\gamma_0 = 20\%$. The Trump tax reform cut the corporate income tax rate to $\gamma_c = 21\%$. This tax rate reduction substantially lowered the effective debt tax shield to $\gamma^* = 2.8\%$. Substituting these values into the bias formula in equation (41) we find

$$N(\widehat{u}) = 8.68 \times N(u).$$

That is, estimated bankruptcy costs here are 8.68 times actual bankruptcy costs. Intuitively, here the firms choose low leverage in rational anticipation of the upcoming tax cut. The econometrician treats the firms as ignorant of the prospective tax cut and treats the low leverage as indicative of very high bankruptcy costs.

Biased parameter estimates will lead to faulty predictions regarding the behavior of firms after the policy change and a faulty assessment of policy tradeoffs. To see this, we continue to follow the parameterization of Goldstein, Ju and Leland (2001), assuming $r = 4.5\%$, $\sigma = 0.25$, $\mu = 0$, and $N(u) = 5\%$. Evaluated at this parameterization, with the equilibrium $\gamma^* = 2.8\%$, equation (38) implies firms observed during the inference stage will choose coupons equal to 13.63% times initial EBIT(=1). Future generations of firms will adopt this same coupon rate. After all, under rational expectations, the inference stage firms posit the same tax shield value as that which will actually be operative post-inference. In other words, no reaction will be apparent when one contrasts the behavior of the inference-stage firms with the behavior of firms post-inference.

The structural econometrician will here mistakenly predict that future generations of firms will respond to the tax rate change by adopting a much lower coupon rate, failing to understand that the inference-stage firms already responded rationally to the upcoming change. In particular, based upon an estimated bankruptcy cost equal to 43.4%(= 8.68 × 5%), equation (38) leads to a predicted coupon rate, call it $\widehat{\phi}$, equal to only 1.49% times initial EBIT. However, as shown above, the actual coupon rate after the tax rate change will be 13.63% times EBIT.

The faulty parameter inference leads to faulty predictions regarding firm behavior after the policy change which in turn leads to a faulty assessment of policy tradeoffs. To illustrate, note that the present value of tax collections per firm in this economy is equal to the value of the perpetual stream of taxes on an unlevered entity minus the tax shield value. It follows that the actual and predicted present value of tax collections are, respectively

$$\begin{aligned} T &= \frac{1-\gamma}{r-\mu} - \gamma\phi^* \left(\frac{1}{r}\right)[1-(\phi^*)^{-\lambda}] = .5546 \\ \widehat{T} &= \frac{1-\gamma}{r-\mu} - \gamma\widehat{\phi} \left(\frac{1}{r}\right)[1-\widehat{\phi}^{-\lambda}] = .6133 \end{aligned}$$

That is, the actual present value of tax collections here will be 10.6% lower than predicted tax collections. Intuitively, the upward bias in estimated bankruptcy costs leads to a faulty prediction of low leverage leading to a faulty prediction of high corporate income tax collections.

6. Multivariate Extension

The preceding sections considered an econometrician attempting to infer one unknown parameter, with the government controlling one policy variable. In this section, we consider a multivariate extension. For simplicity, linearity is assumed.

There are $n_u \geq 1$ unknown deep parameters, each with support on the real line. The realized vector is denoted $\mathbf{u}$. The econometrician seeks to infer $\mathbf{u}$ based upon a vector $\mathbf{m}$ consisting of n_u empirical moments. The government has $n_\gamma \geq 1$ policy tools, with the full-information optimal policy being $\mathbf{g}(\mathbf{u})$.

The observed empirical moments are linear:

$$\mathbf{m} \equiv \mathbf{A}\mathbf{u} + \mathbf{B}\boldsymbol{\gamma}.$$

In the preceding equation, $\mathbf{A}$ is an $n_u \times n_u$ matrix of full rank with element α_{ij} denoting the moment i coefficient on parameter u_j. Matrix $\mathbf{B}$ is an $n_u \times n_\gamma$ matrix with element β_{ij} denoting the moment i coefficient on government policy variable γ_j. The government policy vector is:

$$\boldsymbol{\gamma} = \mathbf{K}\widehat{\mathbf{u}}.$$

In the preceding equation, $\mathbf{K}$ is an $n_\gamma \times n_u$ matrix, with element κ_{ij} denoting the policy i coefficient on $\widehat{u}_j$.

Consider again the nature of bias that arises if the econometrician parameterizes government policy at the status quo

$$\boldsymbol{\gamma}_0 \equiv \mathbf{K}\mathbf{u}_0.$$

The inference equation is

$$\mathbf{A}\mathbf{u} + \mathbf{B}\mathbf{K}\widehat{\mathbf{u}} = \mathbf{A}\widehat{\mathbf{u}} + \mathbf{B}\mathbf{K}\mathbf{u}_0. \tag{43}$$

The left side of the preceding equation is the observed moment assuming real-world firms have rational expectations and the right side is the model-implied moment. Solving the preceding equation we obtain the multivariate analog of equation (21):

$$\begin{aligned} \widehat{\mathbf{u}} &= \mathbf{u} + [\mathbf{A} - \mathbf{B}\mathbf{K}]^{-1}\mathbf{B}\mathbf{K}[\mathbf{u} - \mathbf{u}_0] \\ &= \mathbf{u} + [\mathbf{A} - \mathbf{B}\mathbf{K}]^{-1}\mathbf{B}[\mathbf{g}(\mathbf{u}) - \boldsymbol{\gamma}_0]. \end{aligned} \tag{44}$$

From the preceding equation it follows

$$\mathbf{u} = \mathbf{u}_0 \Rightarrow \mathbf{g}(\mathbf{u}) = \boldsymbol{\gamma}_0 \Rightarrow \widehat{\mathbf{u}} = \mathbf{u}. \tag{45}$$

It follows from the preceding equation that in the multivariate setting $\mathbf{u} = \mathbf{u}_0$ is sufficient for absence of bias, but is not necessary. This is in contrast to the univariate case (Proposition 1) where $u = u_0$ was both necessary and sufficient for absence of bias.

Other implications of the linear multivariate bias equation (44) are most readily illustrated by considering the simplest case with two unknown parameters and one government policy variable. In this case, let β_i denote the moment i coefficient on the government policy variable and let κ_j denote the government policy coefficient on $\widehat{u}_j$. Applying equation (44) we obtain:

$$\begin{aligned} \widehat{u}_1 &= u_1 + \frac{[\beta_1 - \beta_2\alpha_{12}/\alpha_{22}][\kappa_1(u_1 - u_{10}) + \kappa_2(u_2 - u_{20})]}{\alpha_{11} - \beta_1\kappa_1 + [\beta_2\kappa_1\alpha_{12} + \beta_1\kappa_2\alpha_{21} - \beta_2\kappa_2\alpha_{11} - \alpha_{12}\alpha_{21}]/\alpha_{22}} \\ \widehat{u}_2 &= u_2 + \frac{[\beta_2 - \beta_1\alpha_{21}/\alpha_{11}][\kappa_1(u_1 - u_{10}) + \kappa_2(u_2 - u_{20})]}{\alpha_{22} - \beta_2\kappa_2 + [\beta_2\kappa_1\alpha_{12} + \beta_1\kappa_2\alpha_{21} - \beta_1\kappa_1\alpha_{22} - \alpha_{12}\alpha_{21}]/\alpha_{11}}. \end{aligned} \tag{46}$$

With the preceding equation in mind, suppose $\alpha_{12} = \alpha_{21} = 0$. That is, the moment i coefficient on parameter u_j is 0 for $i \neq j$. Here the traditional Jacobian formulation would suggest that the problem of inferring u_1 is separable from the problem of inferring u_2. However, with policy feedback, it is apparent that the inference problems and biases are not separable, since

$$\begin{aligned} \widehat{u}_1 &= u_1 + \frac{\beta_1[\kappa_1(u_1 - u_{10}) + \kappa_2(u_2 - u_{20})]}{\alpha_{11} - \beta_1\kappa_1 - \beta_2\kappa_2/\alpha_{22}} \\ \widehat{u}_2 &= u_2 + \frac{\beta_2[\kappa_1(u_1 - u_{10}) + \kappa_2(u_2 - u_{20})]}{\alpha_{22} - \beta_2\kappa_2 - \beta_1\kappa_1\alpha_{22}/\alpha_{11}}. \end{aligned} \tag{47}$$

Recall also that in the case of one unknown parameter, bias vanishes if: discretionary government policy is not affected by the econometrician's estimate of the parameter ($\kappa = 0$) *or* the moment is not affected by government policy ($\beta = 0$), as shown in equation (21). However, neither of these two conditions is sufficient to eliminate bias in a multivariate setting. To see this, consider again $\alpha_{12} = \alpha_{21} = 0$, and suppose also that the government policy variable does not depend upon $\widehat{u}_2$, with $\kappa_2 = 0$. We then have

$$\begin{aligned} \widehat{u}_1 &= u_1 + \frac{\beta_1\kappa_1(u_1 - u_{10})}{\alpha_{11} - \beta_1\kappa_1} \\ \widehat{u}_2 &= u_2 + \frac{\beta_2\kappa_1(u_1 - u_{10})}{\alpha_{22} - \beta_1\kappa_1\alpha_{22}/\alpha_{11}}. \end{aligned} \tag{48}$$

From the preceding equation it is apparent that even though $\widehat{u}_2$ does not inform policy, $\widehat{u}_2$ will nevertheless be biased so long as the government policy variable influences ($\beta_2 \neq 0$) the respective moment (here m_2) that is relied upon for inferring u_2.

It is also apparent that, in general, the existence of a moment that is independent of the government policy variable does not imply the absence of bias in any particular parameter estimate. To see this, suppose all four elements of matrix $\mathbf{A}$ are positive. Suppose further that the government policy variable has no effect on one of the moments, say m_2, with $\beta_2 = 0$. In this case, bias still emerges, with

$$\begin{aligned} \widehat{u}_1 &= u_1 + \frac{\beta_1[\kappa_1(u_1 - u_{10}) + \kappa_2(u_2 - u_{20})]}{\alpha_{11} - \beta_1\kappa_1 + [\beta_1\kappa_2\alpha_{21} - \alpha_{12}\alpha_{21}]/\alpha_{22}} \\ \widehat{u}_2 &= u_2 + \frac{-[\beta_1\alpha_{21}/\alpha_{11}][\kappa_1(u_1 - u_{10}) + \kappa_2(u_2 - u_{20})]}{\alpha_{22} + [\beta_1\kappa_2\alpha_{21} - \beta_1\kappa_1\alpha_{22} - \alpha_{12}\alpha_{21}]/\alpha_{11}}. \end{aligned} \tag{49}$$

Despite the subtle differences in the nature of bias arising in the univariate and multivariate cases, the solution of the problem is the same: consistent application of rational expectations. To see this, suppose now that the econometrician parameterizes the structural model in a manner consistent with the policies being recommended, with recommended policy $\mathbf{K}\widehat{\mathbf{u}}$ replacing the status quo policy $\mathbf{K}\mathbf{u}_0$ in the original faulty inference equation (43). The rational expectations inference equation is:

$$\mathbf{A}\mathbf{u} + \mathbf{B}\mathbf{K}\widehat{\mathbf{u}} = \mathbf{A}\widehat{\mathbf{u}} + \mathbf{B}\mathbf{K}\widehat{\mathbf{u}} \Rightarrow \widehat{\mathbf{u}} = \mathbf{u}. \tag{50}$$

Conclusion

An asserted advantage of moment-based structural microeconometrics over reduced-form methods is that one can correctly identify policy-invariant parameters so that alternative policy options can be assessed. As we have shown, this approach, which generally treats policy changes as counterfactual zero probability exogenous events, violates rational expectations: agents inside the structural model should understand that policy changes are positive probability endogenous events which the econometric exercise in intended to inform. We examined the implications of this violation of rational expectations in moment-based microeconometric parameter inference which serves a policy function. As shown, bias emerges unless the true value of the parameter justifies the status quo. If instead a policy change is justified, biased inference occurs. Finally, it was shown how rational expectations can be imposed in an internally consistent manner, yielding unbiased inference and optimal policy.

The more general point illustrated by our analysis is that microeconometric methods should vary according to whether the estimation is passive or active in the sense of influencing policy decisions. Although the specifics of the transmission mechanism will differ, the essential problem highlighted by this paper is that with active estimation, future endogenous policy will be correlated with the causal parameters to be estimated. If agents have rational expectations, this channel will bias structural inference if the inference-policy feedback effect is not taken into account. A potentially important direction for future research is to incorporate the policy control channel into the econometric tool-kit, especially as economists get closer to their goal of gaining the attention of policymakers.

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Figure 1: Actual Moment versus Model Moment
Case: m(+,-), g(+)

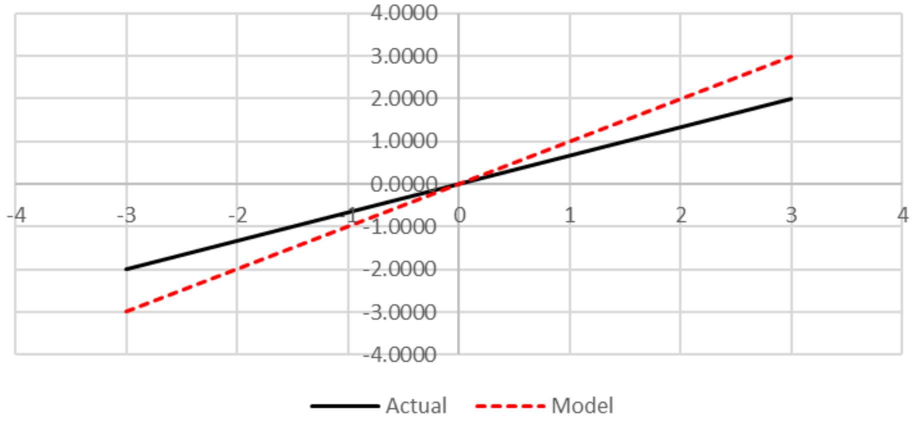


Figure 2: Parameter Inference
Case: m(+,-), g(+)

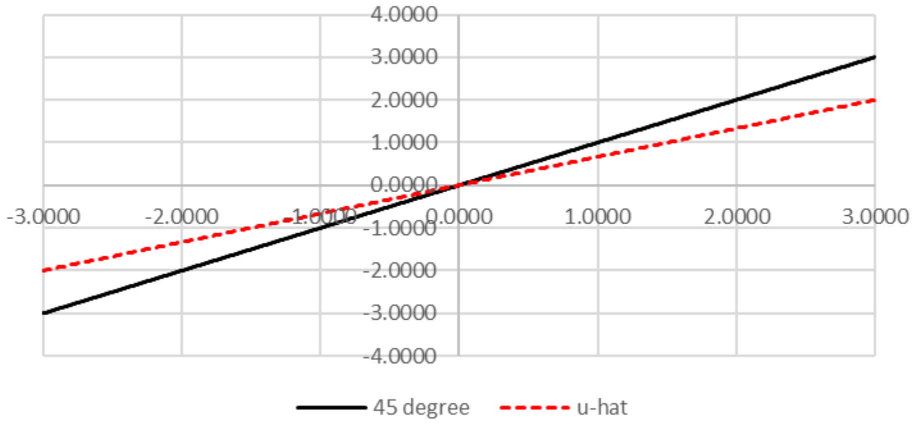


Figure 3: Actual versus Optimal Policy
Case: m(+,-), g(+)

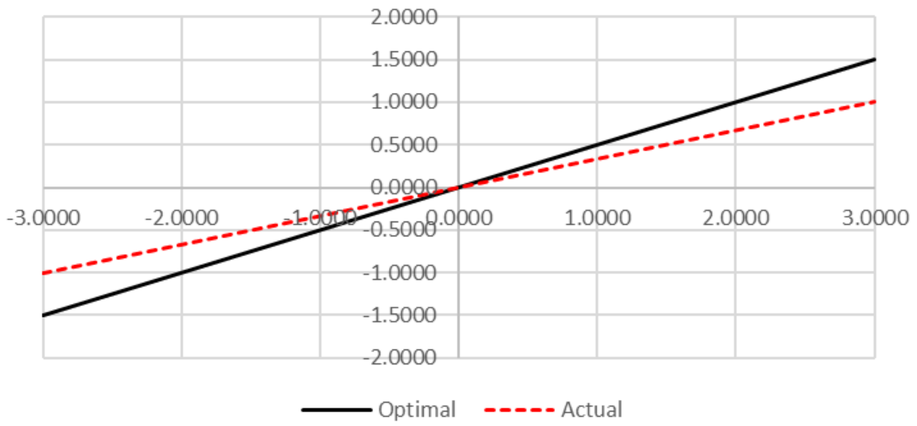


Figure 4: Actual Moment versus Model Moment
Case: m(+,+), g(+)

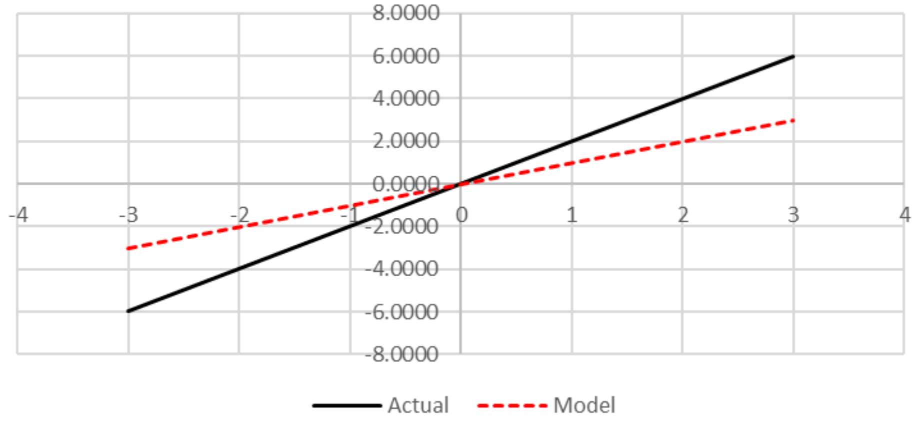


Figure 5: Parameter Inference
Case: m(+,+), g(+)

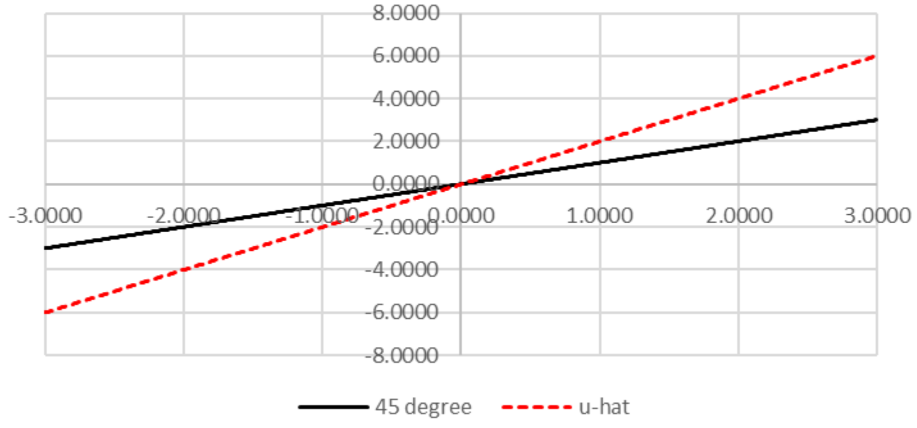
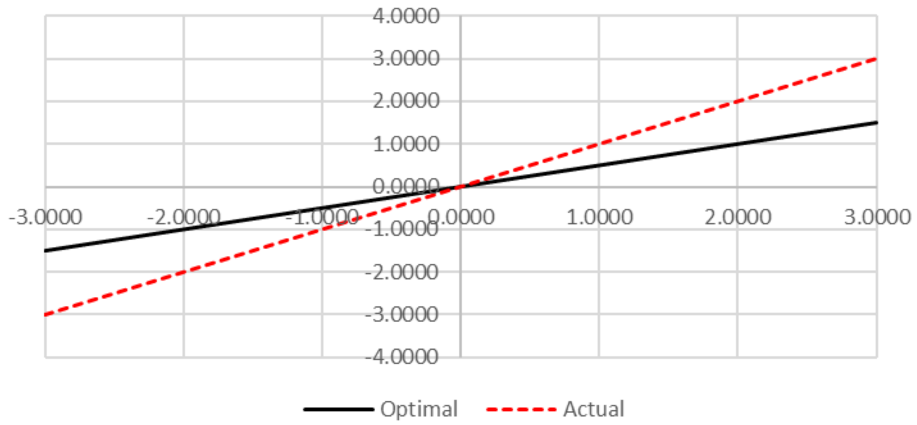


Figure 6: Actual versus Optimal Policy
Case: m(+,+), g(+)



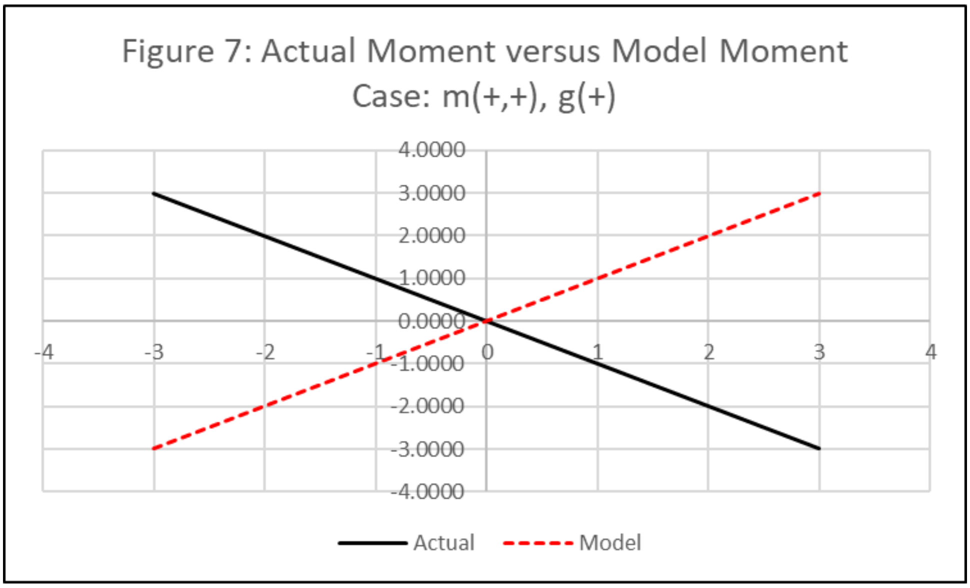


Figure 7: Actual Moment versus Model Moment
Case: m(+,+), g(+)
4.0000
3.0000
2.0000
1.0000
0.0000
-1.0000
-2.0000
-3.0000
-4.0000
-4
-3
-2
-1
0
1
2
3
4
Actual
Model

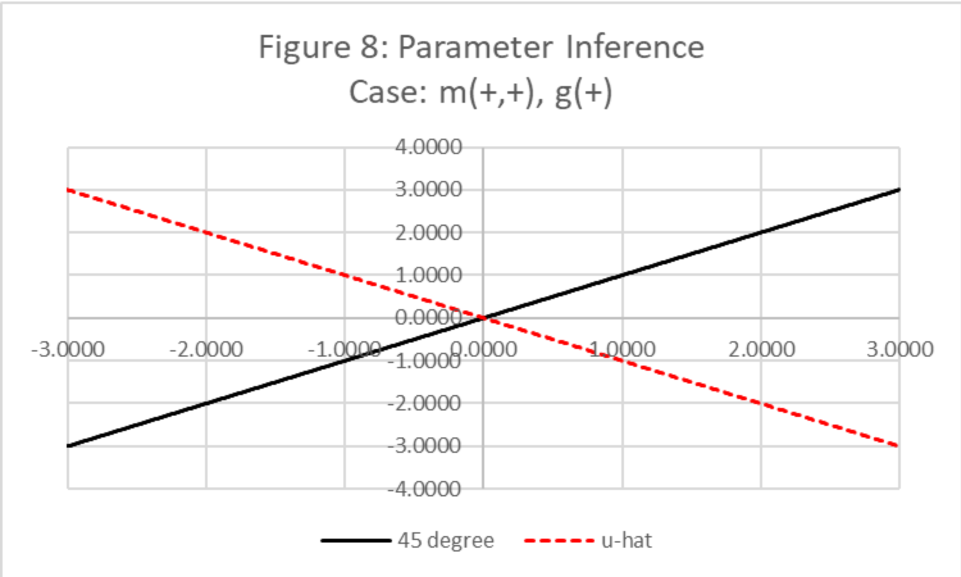


Figure 8: Parameter Inference
Case: m(+,+), g(+)
4.0000
3.0000
2.0000
1.0000
0.0000
-1.0000
-2.0000
-3.0000
-4.0000
-3.0000
-2.0000
-1.0000
0.0000
1.0000
2.0000
3.0000
45 degree
u-hat

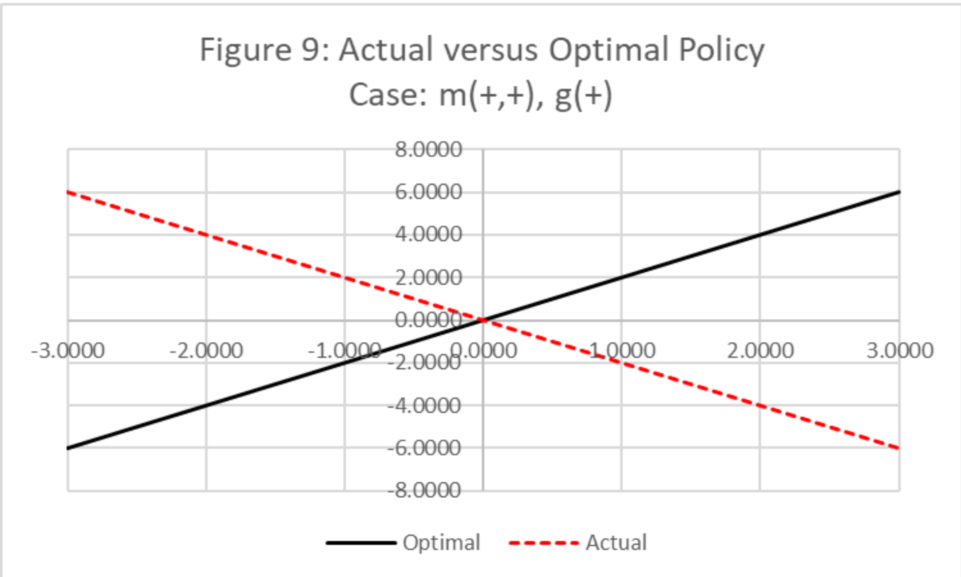


Figure 9: Actual versus Optimal Policy
Case: m(+,+), g(+)
8.0000
6.0000
4.0000
2.0000
0.0000
-2.0000
-4.0000
-6.0000
-8.0000
-3.0000
-2.0000
-1.0000
0.0000
1.0000
2.0000
3.0000
Optimal
Actual