

Sovereign credit risk and exchange rates: Evidence from CDS quanto spreads

November 10, 2017
Conference draft

Abstract

The term structure of sovereign quanto spreads – the difference between CDS premiums denominated in U.S. dollar and a foreign currency corresponding to different maturities– tell us how financial markets view the likelihood of a foreign default and associated currency devaluations at different horizons. A no-arbitrage model can disentangle the two events and associated risk premiums. We study countries in the Eurozone because their quanto spreads pertain to the same exchange rate, and yet their term structures are different. There is a substantial cross-sectional variation in default probabilities leading to variation in term structures in the short-to-medium run. These defaults affect the expected depreciation rate: the risk premium for the Euro devaluation in case of default ranges from a low of 0.06% a week at short and long horizons to a high of 0.45% at the 2-year horizon. Risk-neutral expected depreciation rate conditional on default is only a fraction of relative quanto spreads.

JEL Classification Codes: C1, E43, E44, G12, G15.

Keywords: credit default swaps, quanto spreads, exchange rates, credit risk, sovereign debt, term structure, affine modeling.

1 Introduction

The risk of sovereign default and exchange rate fluctuations are inextricably linked to each other. Depreciation of the home currency often reflects poor economic conditions. Default events tend to be associated with currency devaluations. Such devaluations could either be strategically supporting the competitiveness of the domestic economy, or penalizing a country's growth due to increased borrowing costs or reduced access to international capital markets. Sovereign credit default swaps (CDS) associated with one country but denominated in different currencies offer a window into how financial markets view this interaction. The difference in CDS premiums denominated in different currencies, a.k.a. the CDS quanto spread, delivers the term structure of market expectations regarding the likelihood of default and the associated currency devaluation.

Quanto spreads obtained from U.S. dollar (USD) and Euro (EUR) denominated CDS premiums on Eurozone countries are particularly intriguing. First, quanto spreads reflect market expectations about the USD/EUR exchange rate (FX) regardless of the country. Yet, average term structures of spreads vary in the cross-section. See Figure 1A. They are upward and downward sloping, suggesting that they might be persistent differences affecting the long run. Second, during the period of quanto spread data availability, 2010-2016, the exchange rate itself does not appear to be particularly interesting. See Figure 1B. Despite the famous recalcitrance of depreciation rates, the USD/EUR one is very close to a normally distributed iid series. As we show, such a behavior would imply a flat term structure of quanto spreads, contradicting the evidence.

In this paper, we develop a no-arbitrage model that allows us to reconcile this evidence and use it to extract market expectations about the risks of sovereign default and currency devaluation. These risks are peso events, so one would expect to see little manifestation of such risks in the realized FX series. Intuitively, the cross-sectional differences in term structures of quanto spreads tell us about the differences in the perceived timing of credit events. The modeling challenge is to see whether a parsimonious setup can quantitatively capture all the evidence.

The model we propose features the following critical components: a model of the U.S. reference interest rate curve, a model of credit risk, and a model of FX rate. We use overnight indexed swap (OIS) rates as a reference curve and construct a two-factor model to capture its dynamics.

The starting point for our credit risk model is a credit event arrival that is controlled by a doubly-stochastic Cox process, a popular modeling device in the literature. Default intensity in each country is controlled by two factors – global and regional – with each country having a different set of weights on these factors. We identify the global factor by setting Germany's weights on regional factors to zero. Given that countries in our sample are from the Eurozone during the sovereign debt crisis, we modify our model to allow for a

possibility of credit contagion: a credit event in one country affecting the probability of a credit event in another country.

Last but not least, we model the behavior of the USD/EUR FX rate. We follow the literature on realistic modeling of the time series on FX rates. We allow for time-varying expected depreciation rate, heteroscedastic regular shock to the rates, and for extreme events. We connect jumps in the FX rate to sovereign credit risk by requiring them to take place simultaneously with credit events.

Given these building blocks, we can determine model-based quanto spreads. The data would be informative about the risk-neutral distribution of credit events and currency jumps. We have to confront a typical problem of the credit risk modeling: due to the rarity of credit events, it is difficult to identify the actual distribution of such events. For this reason, it is difficult to measure credit risk premiums.

We confront this challenge in two ways. First, the aforementioned link between credit events and jumps in observed FX rate helps. However, a relatively short sample makes it hard to identify the possible time variation in the actual default intensity via this channel. Thus, as a second leg, we follow [Bai, Collin-Dufresne, Goldstein, and Helwege \(2015\)](#) and associate credit events with extreme movements in quanto spreads.

We estimate the model using joint data on the term structure of quanto spreads of the six countries in Figure 1A, the spot FX rate, and a cross-section of credit events for the six countries to estimate the model via Bayesian MCMC. The model offers an accurate fit to the data. We find that a more parsimonious model without contagion fits the data just as well and does not differ significantly from the larger model in terms of its likelihood. Therefore, we perform the rest of the analysis using the simpler model, which features only four factors: three credit factors (one global, and two regional) and one FX variance factor.

We find that the credit risk of the countries in our sample loads differently on the driving factors. This feature leads to a cross-sectional variation in default hazard rates. In particular, they have different persistence and, therefore, they converge to steady-state values at different speeds. This is why, we observe different term structures in Figure 1A. The steady-state levels that the term structures converge to are in fact the same consistent with the intuition that, in the long run, the Eurozone countries must be in a similar state with respect to default. Germany is the only exception with a 30% lower level of long term quanto spreads, reflecting our strategy for identifying the global credit factor.

The estimated model of the FX rate shows a mild degree of non-normality consistent with the evidence. However, risk-adjusted dynamics exhibit large average devaluations of the EUR conditional on default, 6% (vs less than 1% under the actual probability). Because the expected default timing is different in various countries, these countries are effectively facing a different FX behavior, despite the same currency. Again, these differences wash away in the long run.

We use our model to illustrate the interaction between the FX rate behaviour and default risk. We compare prices and expected payoffs of two types of contracts. The first one is a regular forward contract on the FX rate. The second also pays off the new FX rate at maturity or at default, whichever comes first. Expected cashflows reflect anticipated Euro devaluation. A substantive premium for devaluation in case of default is evident. The difference in excess returns between the two contracts ranges between 6 and 45 basis points per week, depending on horizon.

We also compare relative quanto spreads that are used to estimate the risk-neutral expectation of the future depreciation rate, with the latter object computed from our model. Both relative quanto spreads and risk-neutral expectations exhibit time-variation and pronounced term-structure patterns. The two seem to be closer to each other, on average, at longer maturities with full convergence for Germany at the 15-year horizon. For other countries the maximum percentage contributed by the expected depreciation rate ranges between 40 and 60%. The differences reflect the interaction between the default event and the depreciation rate, conditional on default.

Related literature

This paper is related to two strands of literature. First, it is most closely related to the literature on the relationship between sovereign credit and currency risks. Second, it builds on the vast literature about no-arbitrage affine term structure modeling, and credit-sensitive instruments, prominently summarized in [Duffie and Singleton \(2003\)](#). In particular, it contributes to the literature on the pricing of sovereign CDS, and on the impact of contagion on credit risk. We briefly review the key related papers below, some of which straddle more than one of these areas. For brevity, we focus our attention on the intensity-based credit literature.

Our paper belongs to the strand of the literature that studies interactions between sovereign credit and exchange rate risks. [Corte, Sarno, Schmeling, and Wagner \(2016\)](#) show empirically that the common component in sovereign credit risk correlates with currency depreciations and predicts currency risk premia. [Carr and Wu \(2007\)](#) propose a joint valuation framework for sovereign CDS and currency options with an empirical application to Mexico and Brazil. [Du and Schreger \(2016\)](#) study the determinants of local currency risk as a distinct component of foreign default risk. While closely related, our work is conceptually quite different, as the risk in Western European sovereign CDS reflects default risk irrespective of whether it occurs on domestic or foreign debt. We exploit the entire term structure of CDS quanto spreads to pin down time variation in risk premia associated with conditional expectations of exchange rate depreciation. [Buraschi, Sener, and Mengueterk \(2014\)](#) suggest that geographical funding frictions may be responsible for persistent mispricing in emerging market (EM) bonds denominated in EUR and USD. This is unlikely to be an explanation for USD/EUR quanto spread variation in the Eurozone.

The most recent development in joint FX-sovereign risk literature is research on CDS quanto spreads. [Mano \(2013\)](#) proposes a descriptive segmented markets model that is consistent with nominal and real exchange rate depreciation upon an exogenous default trigger. [Na, Schmitt-Grohé, Uribe, and Yue \(2017\)](#) study the joint occurrence of default and devaluation, i.e., the twin Ds ([Reinhart, 2002](#)), in a small open economy setting with endogenous default and downward nominal wage rigidity. [DeSantis \(2015\)](#) uses quanto spreads to construct measures of redenomination risk. No-arbitrage term structure models for quanto spreads are proposed by [Ehlers and Schoenbucher \(2004\)](#) for Japanese corporate CDS, and by [Brigo, Pede, and Petrelli \(2016\)](#) for Italian CDS. [Monfort, Pegoraro, Renne, and Rousselet \(2017\)](#) apply an Epstein-Zin preference-based framework to the pricing of European CDS quanto spreads.

An important distinction from prior work is that we jointly model the exchange rate risk and quanto spreads for the entire term structure of six countries. In addition, under some mild assumptions about credit events, we estimate both the physical and risk-neutral default intensities. As a result, we can discuss implications for time-varying risk premia associated with default risk and expected depreciation risk conditional on default.

Key empirical contributions to no-arbitrage affine term structure modeling include, but are not limited to, applications to the term structure of interest rates ([Dai and Singleton, 2000](#); [Duffee, 2002](#)), exchange rates ([Backus, Foresi, and Telmer, 2001](#)), corporate credit spreads ([Duffee, 1999](#); [Driessen, 2005](#); [Doshi, Ericsson, Jacobs, and Turnbull, 2013](#)), and sovereign credit spreads ([Duffie, Pedersen, and Singleton, 2003](#); [Hoerdahl and Tristani, 2012](#); [Monfort and Renne, 2013](#)). With respect to the valuation of sovereign CDS, the early affine term structure models have focused on country-by-country estimations, such as Turkey, Brazil, and Mexico ([Pan and Singleton, 2008](#)), Argentina ([Zhang, 2008](#)), or a panel of emerging ([Longstaff, Pan, Pedersen, and Singleton, 2011](#)) or global countries ([Doshi, Jacobs, and Zurita, 2017](#)). [Ang and Longstaff \(2013\)](#) extract a common systemic factor across Europe and the U.S. using sovereign CDS written on European countries and U.S. states, while [Ait-Sahalia, Laeven, and Pelizzon \(2014\)](#) study pairwise contagion among pairs of seven European countries during the sovereign debt crisis.

Finally, we use a model of contagion, which has been an active topic in the recent credit risk literature. [Bai, Collin-Dufresne, Goldstein, and Helwege \(2015\)](#) emphasize that contagion should be an important component of credit risk pricing models in the context of a large number of corporate names. [Benzoni, Collin-Dufresne, Goldstein, and Helwege \(2015\)](#) offer evidence of contagion risk premiums in sovereign CDS spreads in the context of ambiguity-averse economic agents. [Ait-Sahalia, Laeven, and Pelizzon \(2014\)](#) find evidence of contagion under risk-neutral probability in sovereign CDS spreads. [Azizpour, Giesecke, and Schwenkler \(2017\)](#) find evidence of contagion in a descriptive model of realized corporate defaults. We study contagion both under the risk-neutral and objective probabilities.

Table 1 summarizes the specific modeling elements across the key studies with affine intensity-based frameworks for sovereign credit spreads. This table visually highlights the primary

differences between the extant and the current studies. Our work encompasses most of the existing approaches.

2 Sovereign CDS contracts

2.1 Institutional background

Sovereign CDS are contracts that pay off in case of a sovereign credit event. This section reviews what such an event represents. Given the focus on USD/EUR quantos of Eurozone countries, we limit the discussion to legal details associated with European contracts. CDS contracts are controlled by three documents: the 2014 Credit Derivatives Definitions (“the 2014 Definitions”), the ISDA Credit Derivatives Physical Settlement Matrix (“the Physical Settlement Matrix”), and the Confirmation Letter (“the Confirmation”).

The Physical Settlement Matrix is the most important document because the push for standardization has created specific transaction types that are by convention applicable to certain types of a sovereign reference entity, e.g., Standard Western European Sovereign (SWES) or Standard Emerging European Sovereign (SEES) single-name contracts. In total, there are nine transaction types listed in the sovereign section of the Physical Settlement Matrix that contain details about the main contractual provisions for transactions in CDS referencing sovereign entities.

Given the over-the-counter (OTC) nature of CDS contracts, parties are free to combine features from different transaction types, which would be recognized in the Confirmation, i.e., the letter that designates the appropriate terms for a CDS contract. The Confirmation, which is mutually negotiated and drafted between two counterparties, can thereby amend legal clauses attributed to conventional contract characteristics. Hence, there may be slight variations from standard transaction types if counterparties agree to alter the terms of conventional CDS contracts. Such changes introduce legal risk, and potentially make the contracts less liquid, given the customization required for efficient central clearing.

The terms used in the documentation of most credit derivatives transactions are defined in the 2014 Definitions, which update the 2003 Credit Derivatives Definitions.

It is important to distinguish between the circumstances under which a CDS payout/credit event could be triggered, and the restrictions on obligations eligible for delivery in the settlement process upon the occurrence of a qualifying credit event. The Physical Settlement matrix lays out the credit events that trigger CDS payment, which follows the ruling by a determinations committee of the occurrence of a credit event and a credit event auction. SWES transaction types recognize three sovereign credit events, namely Failure to Pay, Repudiation/Moratorium, and Restructuring. SEES contracts further list Obligation Acceleration as a valid event that could trigger the CDS payout.

The most disputed among all credit events is the Restructuring credit event clause related to a change to the reference obligation that is binding on all holders of the obligation. The most controversial among such changes is the redenomination of the principal or interest payment into a new currency. For the credit event to be triggered, this new currency must be any currency other than the lawful currency of Canada, Japan, Switzerland, the United Kingdom, and the United States of America, and the Euro (or any successor currency to any of the currencies listed; in the case of the Euro, the new currency must replace the Euro in whole). In the 2003 Definitions, permitted currencies were defined as those of G7 countries and AAA-rated OECD economies.

Similarly, a redenomination out of the Euro will not be considered a restructuring if the redenomination happens as a result of actions of a governmental authority of an EU Member State, the old denomination can be freely converted into the Euro at the time the redenomination occurred, and there is no reduction in the rate/amount of principal, interest, or premium payable. This means, for example, that if Greece were to exit from the Eurozone, and adopt the new Drachma, it would have to selectively redenominate some of its bonds, and the currency would have to be not freely available for conversion to the Euro for such a credit event to be triggered.

Another important dimension to consider is the obligation category and the associated obligation characteristics which may trigger a credit event. For SWES contracts, the Obligation category is defined broadly as “borrowed money,” which includes deposits and reimbursement obligations arising from a letter of credit or qualifying guarantees. Such contracts also feature no restrictions on the characteristics of obligations relevant for the triggers of default payment. For SEESs, however, markets have agreed on more specificity for the reference obligations, which encompass only “bonds,” which are not allowed to be subordinated, denominated in domestic currency, issued domestically or under domestic law, as indicated by the restrictions of the obligation characteristics.

The final non-trivial aspect relates to the deliverable obligation categories and the associated characteristics. While in the presence of Credit Events for SWESs, bonds or loans are deliverable during the auction settlement process, SEES contracts allow only for bonds to be delivered. Several restrictions apply to the deliverable obligations, such that for SWESs they have to be denominated in a specified currency (i.e., the Euro or the currencies of Canada, Japan, Switzerland, the UK, or the USA), they must be non-contingent, non-bearer and transferable, limited to a maximum maturity of 30 years, and loans must be assignable and consent is required. SEES contracts exclude these restrictions on loans and the maximum deliverable maturity, but impose the additional constraints that the obligation cannot be subordinated, and issued domestically or under domestic law.

We’d like to highlight two implications of these rules that are particularly relevant for this paper. First, a credit event that affects all CDS contracts regardless of the currency of denomination could be triggered by a default pertaining to a subset of bonds, say a sovereign defaulting on domestic but not US-issued bonds. Therefore, a CDS quanto spread would not reflect a risk of selective default. This is in contrast to EM bonds, studied by [Du and](#)

Schreger (2016), where differences between the credit spreads denominated in USD and local currency could be reflecting such a risk.

Second, an obligation is deemed deliverable into the contract settlement regardless of its currency of denomination or that of the CDS contract. This means that one and the same bond, the cheapest-to-deliver one, could be delivered into settlements of CDS contracts of different denominations. Thus, recovery is free of any exchange rate consideration, a point also made by Ehlers and Schoenbucher (2004). Compare this to Mano (2013) who, in the context of EM bonds, explicitly considers different currency denominations of the recovery amount.

2.2 Data

Sovereign CDS contracts became widely available in multiple currencies in 2010. This determines the beginning of our sample, which runs from August 20, 2010 to December 30, 2016. We source daily CDS premiums denominated in USD and EUR from Markit. We use data on the conventionally traded contracts with the full restructuring credit event clause. We require a minimum of 365 days of non-missing information on USD/EUR quanto spreads. This requirement excludes Malta and Luxembourg. Thus, our sample features 17 countries: Austria, Belgium, Cyprus, Estonia, Finland, France, Germany, Greece, Ireland, Italy, Latvia, Lithuania, Netherlands, Portugal, Slovakia, Slovenia, and Spain. We use data on the conventionally traded contracts with the full restructuring credit event clause guided by the 2003 Credit Derivatives Definitions to ensure intertemporal continuity.

We work with weekly data frequency to minimize noise due to potential staleness of some of the price and to maximize the continuity in subsequently observed prices. We have continuous information on 5-year quanto spreads throughout the sample period for all countries, except for Greece, as the trading of its sovereign CDS contract halted following its official default in 2012. In addition, we retain the maturities of 1, 3, 7, 10, and 15 years (we omit the available 30-year contracts because it is similar to the 15-year one, in particular the term structures between 15 and 30 are flat). Even though the 5-year contract is the most liquid, liquidity across the term structure is less of a concern for sovereign CDS spreads compared to corporate CDS spreads, as trading is more evenly spread out across the maturity spectrum (Pan and Singleton, 2008).

Table 2 provides basic summary statistics for the quanto spreads. There is a significant amount of both cross-sectional and time-series variation in the spreads. In the cross-section, the average quanto spread ranges from 6 basis points (bps) for Estonia to 90 bps for Greece, at the 5-year maturity. The average quanto slope, defined as the difference between the 10-year and 1-year quanto spreads, ranges from -29bps for Greece to 29 bps for France. Overall, there is a significant amount of variation over time in both the level and the slope of CDS quanto spreads for each country. Figure 1A complements these statistics by plotting average quanto term spreads of different maturities for a select group of countries.

Even though these sovereign CDS contracts trade in multiple currencies, there might be differences in liquidity, given that an insurance payment in EUR would likely be less valuable if Germany defaults. Consistent with this view, USD denominated contracts tend to be more liquid, as is documented in Table 3, which reports the average number of dealers quoting such contracts in either EUR or USD over time, i.e., CDS depth (Qiu and Yu, 2012). The average difference between USD and EUR ranges between 0.60 to 2.66 dealers. EUR CDS contracts are quoted by 2.73 to 6.30 dealers, on average, which is economically meaningful, given the large concentration of the CDS market among a handful of dealers (Giglio, 2014; Siriwardane, 2014).

Notional amounts outstanding, also reported in Table 3, offer a sense of cross-sectional variation in the size of the market. Regardless of the currency of denomination, the notional are converted into USD and reported on the gross and net basis. To facilitate comparison, we express these numbers as a percent of the respective quantities for Italy that has the largest gross and net notionals (Augustin, Sokolowski, Subrahmanyam, and Tomio, 2016). France, Germany, and Spain stand out as a fraction of Italy’s amounts followed by Austria, Belgium, and Portugal.

To further gauge the size of the market for single-name sovereign CDS, we compare the gross notional amounts outstanding to the aggregate market size. Augustin (2014) reports that in 2012, single-name sovereign CDS accounted for approximately 11% of the overall market, which was then valued at \$27 trillion in gross notional amounts outstanding. The corporate CDS market accounted for about 89% of the market, with single-name and multi-name contracts amounting to \$16 trillion and \$11 trillion, respectively. While the CDS market has somewhat shrunk in recent years, statistics from the Bank for International Settlements suggest that sovereign CDS represented with \$1.715 trillion about 18% of the entire market in 2016.

We collect the time series of the USD/EUR FX rate from the Federal Reserve Bank of St. Louis Economic Database (FRED) and match it with the quanto data, using weekly exchange rates, sampled every Wednesday. As we have emphasized in the introduction, Figure 1B highlights that the distribution of exchange rate movements was close to normal during our sample period.

We also need information on the term structure of U.S. interest rates. Prior to the global financial crisis of 2008/09 (GFC), it was a common practice to use Libor and swap rates as the closest approximation to risk-free lending rates in the interdealer market (Feldhutter and Lando, 2008). After the GFC, practitioners have shifted towards full collateralization and using OIS rates as better proxies of risk-free rates (Hull and White, 2013). This shift has implications for Libor-linked interest rate swaps (IRS) because discounting is performed using the OIS-implied curves. We source daily information on OIS and IRS rates for all available maturities from Bloomberg, focusing on OIS rates with maturities of 3, 6, 9, 12, 36, and 60 months, and IRS rates with maturities of 7, 10, 15, and 30 years.

We bootstrap zero coupon rates from all swap rates. We transform all swap rates into

par-bond yields, assuming a piece-wise constant forward curve, and then extract the zero-coupon rates of the same maturities as the swap rates. Thus, we obtain a zero-coupon yield curve up to five years estimated from OIS rates, and estimated from IRS rates for maturities above five years. In order to extend the OIS zero-curve for maturities beyond five years, we use the zero-coupon yield bootstrapped from IRS rates, but adjusted daily by the differential between the IRS and OIS implied zero-coupon curves. Figure 2 displays the resulting rates.

Our last piece of evidence pertains to the actual occurrence of credit events. Actual default information is insufficient for estimating conditional credit event probabilities because they are rare. This issue is common to the literature on credit-sensitive instruments. When one models corporate defaults, it is possible to say something about actual conditional default probabilities by grouping companies by their credit rating, as was done by [Driessen \(2005\)](#), for instance.

We are considering high quality sovereign names, so we have only the credit event by Greece in our sample. Formal defaults are often avoided because of bailouts, as was witnessed multiple times during the sovereign debt crisis (Greece, Ireland, Portugal, Spain, Cyprus). These bailouts result in large movements in credit spreads, even though no formal credit event occurred. Therefore, we associate credit events with extreme movements in quanto spreads (see, also, [Bai, Collin-Dufresne, Goldstein, and Helwege, 2015](#)).

Specifically, we deem a credit event to have occurred if a weekly (Wednesday to Wednesday) change in the 5-year quanto spread is above the 99th percentile of the country-specific distribution of quanto spread changes. Figure 3 shows observed credit events identified this way for the 16 countries in our sample. We observe, clustering of the events towards the end of 2011 and around 2012. Thus, visual inspection is supportive of the notion of contagion.

Although Greece is the country that experienced a formal credit event, we omit it for pragmatical reasons (see, also, [Ait-Sahalia, Laeven, and Pelizzon, 2014](#)). As, Figure 4A shows, the Greek CDS premium jumped to 5,062 bps on September 13, 2011, long before the actual declaration of the credit event on March 9, 2012. It exceeded the 10,000 bps threshold, that is, an equivalent to 100% of insured face value, on February 15, 2012. Furthermore, trading of Greece CDS spreads even halted between March 8, 2012 and June 10, 2013, such that the time series exhibits a long gap in quoted premiums. Assuming the Markit-reported aggregation of quoted spreads was tradable, Greek CDS started trading again on June 10, 2013, at levels of 978 bps. The corresponding quanto spreads displayed in Figure 4B exhibit similar swings in magnitudes and gaps in the data. These data problems create severe credit risk identification issues and makes it difficult to study joint behavior of credit factors across countries.

3 A model

In this section we present a no-arbitrage model of the joint dynamics of U.S. interest rates, USD/EUR FX rate, and CDS quanto spreads for Eurozone countries. In broad strokes, the key part of the model is the connection between devaluation of the FX rate and sovereign default. Mathematically, we model devaluation via a Poisson jump. To connect devaluation to default, we make a simplifying assumption that jumps in the FX rate can take place only if one of the Eurozone sovereigns experiences a credit event. This assumption links sovereign default hazard rates to the FX Poisson arrival rate. Hazard rates feature one common component, that is linked to Germany, and regional components. Furthermore, we allow for default contagion effects.

3.1 Pricing kernel

Suppose, $M_{t,t+1}$ is the pricing kernel. We can value a cash flow, $\mathcal{X}_{t+1}$, using the pricing kernel via $E_t(M_{t,t+1}\mathcal{X}_{t+1})$, where the expectation is computed under objective conditional probability $p_{t,t+1}$. Alternatively, one can value the same cash flow using the risk-neutral approach

$$E_t(M_{t,t+1}\mathcal{X}_{t+1}) = E_t(M_{t,t+1})E_t\left(\frac{M_{t,t+1}}{E_t(M_{t,t+1})}\mathcal{X}_{t+1}\right) = e^{-r_t}E_t^*(\mathcal{X}_{t+1}),$$

where the expectation is computed under risk-neutral conditional probability $p_{t,t+1}^*$, and r_t is the risk-free rate at time t . Thus the pricing kernel connects the two probabilities via $M_{t,t+1}/E_t M_{t,t+1} = e^{-r_t}p_{t,t+1}^*/p_{t,t+1}$. In this paper, we use both valuation approaches interchangeably.

3.2 Generic CDS valuation

We start with valuation in order to introduce the key objects that we model in subsequent sections. A CDS contract with time to maturity T has two legs. The premium leg pays the CDS premium $C_{t,T}$ every quarter until a default takes place at random time τ . It pays nothing after default. The protection leg pays a fraction of the face value of debt that is lost in default and nothing if there is no default before maturity.

Accordingly, the present value of fixed payments of the USD-denominated contract that a protection buyer pays is

$$\pi_t^{pb} = C_{t,T}^s \sum_{j=1}^{(T-t)/\Delta} E_t[M_{t,t+j\Delta} I(\tau > t + j\Delta)], \quad (1)$$

where $M_{t,t+i}$ is the USD denominated nominal pricing kernel, Δ is the time interval between two successive coupon periods, and $I(\cdot)$ is the indicator function. We have omitted accrual payments for notational simplicity.

A protection seller is responsible for any losses L upon default, which we fix to be a constant in line with the literature on CDS pricing (Pan and Singleton, 2008), and thus the net present value of future payments is given by

$$\pi_t^{ps} = L \cdot E_t[M_{t,\tau} I(\tau \leq T)].$$

The CDS premium $C_{t,T}^{\$}$ is determined by equalizing the values of the two legs.

If S_t represents the nominal USD/EUR FX rate (amount of USD per one EUR), the premium of a EUR-denominated contract is

$$C_{t,T}^{\text{€}} = L \cdot \frac{E_t[M_{t,\tau} I(\tau \leq T) S_{\tau \wedge T}]}{\sum_{j=1}^{(T-t)/\Delta} E_t[M_{t,t+j\Delta} I(\tau > t+j\Delta) S_{t+j\Delta}]}, \quad (2)$$

where $\wedge$ denotes the smallest of the two variables.

These expressions help us to interpret the information conveyed by CDS quanto spreads. To streamline the interpretation, consider a hypothetical contract that trades all points upfront, meaning that a protection buyer pays the entire premium at time t . Assume that the risk-free rate is constant. Then the EUR-denominated CDS premium simplifies to

$$C_{t,T}^{\text{€}} = L \cdot E_t[M_{t,\tau} I(\tau \leq T) S_{\tau \wedge T} / S_t] = L \cdot E_t^*[e^{-r(\tau \wedge T - t)} I(\tau \leq T) S_{\tau \wedge T} / S_t],$$

and the relative quanto spread is

$$\begin{aligned} \frac{C_{t,T}^{\$} - C_{t,T}^{\text{€}}}{C_{t,T}^{\$}} &= \frac{E_t^*[e^{-r(\tau \wedge T - t)} I(\tau \leq T) (1 - S_{\tau \wedge T} / S_t)]}{E_t^*[e^{-r(\tau \wedge T - t)} I(\tau \leq T)]} \\ &= E_t^* \left[1 - \frac{S_{\tau \wedge T}}{S_t} \right] - \text{cov}_t^* \left[\frac{e^{-r(\tau \wedge T - t)} I(\tau \leq T)}{E_t^*[e^{-r(\tau \wedge T - t)} I(\tau \leq T)]}, \frac{S_{\tau \wedge T}}{S_t} \right]. \end{aligned} \quad (3)$$

The first term in (3) reflects risk-neutral expected currency depreciation, conditional on a credit event (positive number corresponds to the EUR devaluation). The covariance term reflects the interaction between the two.

It is helpful to introduce the concepts of survival probabilities and hazard rates to handle computations of expectations involving indicator functions $I(\cdot)$. The information set $\mathcal{F}_t$ includes all the available information up to time t excluding credit events. Let

$$H_t \equiv \text{Prob}(\tau = t \mid \tau \geq t; \mathcal{F}_t)$$

be the conditional instantaneous default probability of a given reference entity at day t , a.k.a., the hazard rate. Further, let

$$P_t \equiv \text{Prob}(\tau > t \mid \mathcal{F}_t)$$

be the time- t survival probability conditional on no earlier default up to and including time t . P_t is related to the hazard rate H_t via

$$P_t = P_0 \prod_{j=1}^t (1 - H_j), \quad t \geq 1. \quad (4)$$

Applying the Law of Iterated Expectations to both the numerator and the denominator, we can rewrite the CDS premium as

$$C_{t,T}^{\epsilon} = L \cdot \frac{\sum_{j=1}^{T-t} E_t[M_{t,t+j}(P_{t+j-1} - P_{t+j})S_{t+j}]}{\sum_{j=1}^{(T-t)/\Delta} E_t[M_{t,t+j\Delta}P_{t+j\Delta}S_{t+j\Delta}]}. \quad (5)$$

A similar expression obtains for the USD-denominated contract by setting $S_t = 1$.

Appendix A shows that the term structure of credit premia is flat if both the hazard and depreciation rates are iid but correlated with each other. This result establishes a useful benchmark for interpreting the evidence summarized in Figure 1A.

3.3 Credit event contagion

Given the focus on default contagion in the recent literature on both corporate and sovereign credit markets, we allow for this feature in our model. To gain intuition about how our model of contagion works, consider Poisson arrival of credit events with a conditional rate d_t . We would like realizations from this process to affect the conditional rate in the subsequent period. Denote the realization by $\mathcal{P} : \mathcal{P}|d_t \sim \text{Poisson}(d_t)$.

In our application to Eurozone sovereigns, we expect d_t to be small implying most of realizations of $\mathcal{P}$ to be equal to zero (probability of such an event is e^{-d_t}). Occasionally, with probability $d_t e^{-d_t}$, there is going to be a single event. Theoretically, it is possible that $\mathcal{P} > 1$ with the probability $1 - e^{-d_t} - d_t e^{-d_t}$. But for a small d_t , such an outcome is unlikely.

In this respect, such a Poisson process can be viewed as an analytically tractable approximation to a Bernoulli distribution that is more appropriate for a credit event in a single country. For parsimony reasons, we use this process to count all contemporaneous events across the countries in our sample. Thus, a Poisson model is a better fit for our framework.

The next step of the contagion model is to determine how the value of $\mathcal{P}$ affects the subsequent arrival rate d_{t+1} . First, this value has to be non-negative, so we choose a distribution with a non-negative support. Second, we would like to achieve analytical tractability for valuation purposes, so we choose a Gamma distribution whose shape parameter is controlled by $\mathcal{P} : d_{t+1} \sim \text{Gamma}(\mathcal{P}, 1)$. The idea is that the more credit events we have at time t , the larger is the impact on d_{t+1} . If $\mathcal{P}|d_t = 0$, then $d_{t+1} = 0$, by convention.

The resulting distribution of d_{t+1} is

$$\phi(d_{t+1} | d_t) = \sum_{k=1}^{\infty} \left[\frac{d_t^k}{k!} e^{-d_t} \times \frac{d_{t+1}^{k-1} e^{-d_{t+1}}}{\Gamma(k)} \right] \mathbf{1}_{[d_{t+1} > 0]} + e^{-d_t} \mathbf{1}_{[d_{t+1} = 0]}.$$

This expression represents the description in words above. It makes explicit what is missing. We have to replace d_t in this expression by $\bar{d} + \phi d_t$. The constant is needed to preclude $d_t = 0$ from becoming an absorbing state. The coefficient $0 < \phi < 1$ is needed to ensure stationarity of d_t . Such a model happens to be autoregressive gamma-zero, $ARG_0(\bar{d} + \phi d_t, 1)$, a process introduced by [Monfort, Pegoraro, Renne, and Rousselet \(2014\)](#) for the purpose of modeling interest rates at the zero lower bound. In our model, the contagion factor d_t interacts with other factors controlling credit risk as described below.

[Monfort, Pegoraro, Renne, and Rousselet \(2017\)](#) use ARG_0 to model credit contagion as well. [Ait-Sahalia, Laeven, and Pelizzon \(2014\)](#) and [Azizpour, Giesecke, and Schwenkler \(2017\)](#) model contagion with a similar Hawkes process. [Benzoni, Collin-Dufresne, Goldstein, and Helwege \(2015\)](#) consider a different mechanism where contagion arrives via learning about a hidden state. [Bai, Collin-Dufresne, Goldstein, and Helwege \(2015\)](#) rely on multitude of firms in developing a model of contagion, and, therefore, it is not readily applicable to the sovereign sector.

3.4 Initial setup

States

We assume that if investors were risk-neutral, then an $N \times 1$ -dimensional multivariate state vector x_{t+1} would evolve according to

$$x_{t+1} = \mu_x^* + \Phi_x^* x_t + \Sigma_{x,t}^* \cdot \varepsilon_{x,t+1},$$

where $\varepsilon_{x,t+1}$ defines a vector of independent shocks, Φ_x^* is a $N \times N$ matrix with positive diagonal elements, $\Sigma_{x,t}^*$ is a $N \times N$ matrix that is implied by the specification described below. The state x_t consists of three different sub-vectors

$$x_t = (u_t^\top, g_t^\top, d_t^\top)^\top.$$

U.S. interest rate curve

The factor u_t is a $M_u \times 1$ vector that follows a Gaussian process:

$$u_{t+1} = \mu_u^* + \Phi_u^* u_t + \Sigma_u \cdot \varepsilon_{u,t+1},$$

and $\varepsilon_{u,t+1} \sim \mathcal{N}(0, I)$, μ_u^* is an $M_u \times 1$ vector, and Φ_u^* and Σ_u are all $M_u \times M_u$ matrices, and the diagonal elements of Σ_u are denoted by σ_{u_i} , for $i = 1, 2, \dots, M_u$.

The default-free U.S. dollar interest rate (OIS swap rate) is

$$r_t = \bar{r} + \delta_u^\top u_t. \quad (6)$$

In the applications, we assume for simplicity that there are only $M_u = 2$ interest rate factors such that $u_t = (u_{1,t}, u_{2,t})^\top$.

Credit risk

The factor g_t is an autonomous multivariate autoregressive gamma process of size M_g . Each component $i = 1, \dots, M_g$ follows an autoregressive gamma process, $g_{i,t+1} \sim ARG(\nu_i, \phi_i^{*\top} g_t, c_i^*)$, that can be described as

$$g_{i,t+1} = \nu_i c_i^* + \phi_i^{*\top} g_t + \eta_{i,t+1},$$

where ϕ_i^* is a $M_g \times 1$ vector, and $\eta_{i,t+1}$ represents a martingale difference sequence (mean zero shock) with conditional variance given by

$$\text{var}_t \eta_{i,t+1} = \nu_i c_i^{*2} + 2c_i^* \phi_i^{*\top} g_t$$

with $c_i^* > 0$ and $\nu_i > 0$ that define the scale parameter and the degrees of freedom, respectively. The multivariate autoregressive gamma process requires the parameter restrictions $0 < \phi_{ii}^* < 1$, $\phi_{ij}^* > 0$, for $1 \leq i, j \leq M_g$. See [Gourieroux and Jasiak \(2006\)](#) and [Le, Singleton, and Dai \(2010\)](#). We further separate the factor g_t into factors w_t and v_t that will be used for modeling the credit and currency risks, respectively.

The default hazard rate of each country $k = 1, \dots, M_c$ is $H_t^{*k} = P^*(\tau^k = t | \tau^k \geq t; \mathcal{F}_t)$, where τ^k is the credit event time in country k , and M_c is the number of countries. We posit that the hazard rate is determined by the default intensity h_t^{*k} as follows:

$$H_t^{*k} = 1 - e^{-h_t^{*k}}, \quad h_t^{*k} = \bar{h}^{*k} + \delta_w^{*k\top} w_t + \delta_d^{*k\top} d_{t-1}, \quad (7)$$

such that the default intensity is affine in the state variables. We assume that w_t consists of G global and K regional factors, so that each intensity h_t^{*k} would be a function of all global factors and one of the regional factors. We have introduced the contagion factor d_t in section 3.3. We elaborate on the details in the sequel.

We assume $G = 1$ and $K = 2$ in our empirical work, implying two factors per country (one global and one regional; by assumption Germany is exposed to the global factor only as in [Ang and Longstaff, 2013](#)). This choice is motivated by a principal component analysis (PCA) that extracted country-specific components from the quanto spreads. The procedure implies that two factors explain around 99% of the variation in quanto spreads. Further,

the PCA of the combination of the first two components across all countries implies that the first principal component explains 58% of the variation.

Furthermore, as the number of parameters grows with the number of countries, we limit the model estimation to six sovereigns. We choose the countries that exhibit the greatest market liquidity and the least amount of missing observations. In addition, we incorporate both peripheral and core countries that feature the greatest variation in the average term structure of CDS quanto spreads. This leads us to focus on Germany, Belgium, France, Ireland, Italy, and Portugal. These are the countries displayed in Figure 1A.

Default contagion

The final factor d_t is a multivariate autoregressive gamma-zero process of size M_d . Each component $k = 1, \dots, M_d$ follows an autoregressive gamma-zero process, $d_{t+1}^k \mid w_{t+1} \sim ARG_0(\bar{h}^{*k} + \delta_w^{*k\top} w_{t+1} + \delta_d^{*k\top} d_t^k, \rho^{*k})$. Compared to the description in section 3.3, we add two more features. First, the contagion factor is affected by conventional credit factors w_t in addition to its own value from the previous period. Second, we allow for a scale parameter, ρ^{*k} , that could be different from unity in the Gamma distribution.

Besides the explicit distribution, an ARG_0 process can be described as

$$d_{t+1}^k = \bar{h}^{*k} + \delta_w^{*k\top} w_{t+1} + \delta_d^{*k\top} d_t^k + \eta_{t+1}^k, \quad (8)$$

where $\eta_{k,t+1}$ is a martingale difference sequence (mean zero shock), with conditional variance given by

$$\text{var}_t \eta_{t+1}^k = 2\rho^{*k} \left(\bar{h}^{*k} + \delta_w^{*k\top} \left[\nu_w \odot c_w^* + \phi_w^{*\top} w_t \right] + \delta_d^{*k\top} d_t \right),$$

where $\odot$ denotes the Hadamard product. Following [Le, Singleton, and Dai \(2010\)](#), we impose the following parameter restrictions $0 < \delta_{d_{ii}}^{*k} < 1$, $\delta_{w_{ij}}^{*k}, \delta_{d_{ij}}^{*k} > 0$, for $1 \leq i, j \leq M_w, M_d$. Comparing expressions (7) and (8) makes it clear that the default hazard rate and arrival rate of Poisson events in the contagion factors are the same process.

For parsimony, we assume the existence of one common credit event variable that may induce contagion across the different countries and regions. This is conceptually similar to [Benzoni, Collin-Dufresne, Goldstein, and Helwege \(2015\)](#), who suggest that a shock to a hidden factor may lead to an updating of the beliefs about the default probabilities of all countries. Thus, given such a restriction, the contagion factor is a scalar, $d_{t+1} \mid w_{t+1} \sim ARG_0(\bar{h}^* + \delta_w^{*\top} w_{t+1} + \delta_d^{*\top} d_t, \rho^*)$ with appropriate restrictions on loadings:

$$\bar{h}^* = \sum_k \bar{h}^{*k}, \quad \delta_w^* = \sum_k \delta_w^{*k}, \quad \delta_d^* = \sum_k \delta_d^{*k}.$$

As a result we may have more than one credit event per period.

FX rate

We model the foreign exchange rate S_t as the amount of USD per one EUR. The idea of our model is that the (log) depreciation rate should be a linear function of the state and be exposed to two additional shocks. One is currency-specific normal shock with varying variance v_t , another one is an extreme move associated with devaluation. We posit:

$$\Delta s_{t+1} = \bar{s}^* + \delta_s^{*\top} x_{t+1} + (\bar{v} + \delta_v^\top v_t)^{1/2} \cdot \varepsilon_{s,t+1} - z_{s,t+1},$$

where $\varepsilon_{s,t+1} \sim \text{Normal}(0, 1)$ and is independent of $\varepsilon_{x,t+1}$. We assume the variance factor v_t to be one-dimensional. A jump z_{t+1} is drawn from an independent Poisson-Gamma mixture distribution. Specifically, the jump arrival rate j_{t+1} follows a Poisson distribution with intensity λ_{t+1}^* , $j_{t+1} \sim \mathcal{P}(\lambda_{t+1}^*)$, and $z_{s,t+1} | j_{t+1} \sim \text{Gamma}(j_{t+1}, \theta^*)$. The minus sign in front of z_s emphasizes that EUR devalues in case of a Eurozone's sovereign credit event.

Jumps in the FX rate are linked to sovereign default risk because of our assumption that FX jumps only in case of a credit event. Therefore, the FX jump intensity is equal to the sum of all country-specific default intensities:

$$\lambda_t^* = \sum_k h_t^{*k} = \sum_k \bar{h}^{*k} + \sum_k \delta_w^{*k} \cdot w_t + \sum_k \delta_d^{*k} \cdot d_{t-1} = E^*(d_t | w_t, d_{t-1}).$$

This model could be equivalently written as:

$$\Delta s_{t+1} = \bar{s}^* + \delta_s^{*\top} \mu_x^* + \delta_s^{*\top} \Phi_x^* x_t + \delta_s^{*\top} \Sigma_x^* \cdot V_t^{1/2} \cdot \varepsilon_{x,t+1} + (\bar{v} + \delta_v^\top v_t)^{1/2} \cdot \varepsilon_{s,t+1} - z_{s,t+1}.$$

This expression highlights the (risk-neutral) expected depreciation rate, $\bar{s}^* + \delta_s^{*\top} \mu_x^* + \delta_s^{*\top} \Phi_x^* x_t$, and the fact that depreciation rate can be conditionally and unconditionally correlated with states x_t . The model is more parsimonious than the most general one (loadings δ_s^* control both expectations and innovations). This expression also shows that we can explore the question of whether regular innovations or jumps in the depreciation rate contribute to the magnitude of quanto spreads the most (see [Brigo, Pede, and Petrelli, 2016](#); [Carr and Wu, 2007](#); [Ehlers and Schoenbucher, 2004](#); [Monfort, Pegoraro, Renne, and Rousselet, 2017](#) for related discussions).

3.5 Valuation of securities in the model

Prices of risk

We have articulated all the modeling components that are needed for security valuation. In order to estimate the model, we need the behavior of state variables under the objective probability of outcomes. [Appendix B](#) demonstrates that, there exists a pricing kernel that supports a flexible change in the distribution of variables involved in valuation of securities.

Most parameters could be different under the two probabilities. One may recover evolution of state variables under the objective probability by dropping asterisks $*$ in the expressions of section (3.4).

Given the focus on credit events, we would like to highlight how prices of default risk work in our model. All the variables that are related to credit events have objective, h_t^k , λ_t , and risk-neutral, h_t^{*k} , λ_t^* , versions because the event risk premium could be time varying. In particular, the actual and risk-neutral counterparts may have a different functional form and a different factor structure. In addition, each of these variables may have different actual and risk-neutral distributions that are related to the respective distributions of the factors driving them.

Risk-adjusted and actual distributions of h_t^{*k} and λ_t^* can be identified from the cross-section and time series of quanto spreads, respectively. The actual event frequencies h_t^k and λ_t can be identified only from realized credit events themselves – a challenge for financial assets of high credit quality. As mentioned earlier, we circumvent this difficulty, by associating credit events with extreme movements in quanto spreads. Even in this case the empirical problem is quite challenging, so we only model common events and assume that they are directed by the same factors as their risk-neutral counterparts: $d_t \mid w_t \sim ARG_0(\bar{h} + \delta_w^\top w_t + \delta_d d_{t-1}, \rho)$, and $\lambda_t = E(d_t \mid w_t, d_{t-1})$.

U.S. OIS swap rates

The price of a zero-coupon bond paying one unit of the numeraire n -periods ahead from now satisfies

$$Q_{t,T} = E_t^* B_{t,T-1}, \quad (9)$$

where $B_{t,t+j} = \exp(-\sum_{u=0}^j r_{t+u})$. Given the dynamics of the interest rate defined in equation (6), bond prices can be solved using standard techniques such that log zero-coupon bond prices q_t are affine in the interest rate state variables u_t , such that the term structure of interest rates is given by

$$y_{t,T} \equiv -(T-t)^{-1} \log Q_{t,T} = A_{T-t} + B_{T-t}^\top u_t. \quad (10)$$

See Appendix C.

CDS

We can express the CDS spread presented in equation (5) using the risk-neutral probability as

$$C_{t,T}^\epsilon = L \cdot \frac{\sum_{j=1}^{T-t} E_t^*[B_{t,t+j-1}(P_{t+j-1}^* - P_{t+j}^*)S_{t+j}]}{\sum_{j=1}^{(T-t)/\Delta} E_t^*[B_{t,t+j\Delta-1}P_{t+j\Delta}^*S_{t+j\Delta}]}. \quad (11)$$

We can use recursion techniques to derive analytical solutions for CDS premiums by solving for the following two objects

$$\tilde{\Psi}_{j,t} = E_t^* \left[B_{t,t+j-1} \frac{P_{t+j-1}^*}{P_t^*} \frac{S_{t+j}}{S_t} \right] \quad \text{and} \quad \Psi_{j,t} = E_t^* \left[B_{t,t+j-1} \frac{P_{t+j}^*}{P_t^*} \frac{S_{t+j}}{S_t} \right], \quad (12)$$

These expressions jointly yield the solution for the CDS premium after dividing the numerator and the denominator of equation (11) by the time- t survival probability P_t^* and exchange rate S_t .

$$C_{t,T}^\epsilon = L \cdot \frac{\sum_{j=1}^{T-t} (\tilde{\Psi}_{j,t} - \Psi_{j,t})}{\sum_{j=1}^{(T-t)/\Delta} \Psi_{j\Delta,t}}. \quad (13)$$

To evaluate the expressions for $\tilde{\Psi}$ and Ψ , we conjecture that the expressions in equation (12) are exponentially affine functions of the state vector x_t :

$$\tilde{\Psi}_{j,t} = e^{\tilde{A}_j + \tilde{B}_j^\top x_t} \quad \text{and} \quad \Psi_{j,t} = e^{A_j + B_j^\top x_t}. \quad (14)$$

See Appendix D for the derivation of these loadings.

3.6 Implementation

We use data on the USD/EUR FX rate, the term structure of OIS interest rates, the term structures of CDS quanto spreads for a cross-section of six countries, and a cross section of credit events for six countries to estimate the model. The model is estimated using Bayesian MCMC. See Appendix E. The output of the procedure are the state variables and parameter estimates. We outline parameter restrictions that we impose.

We make the following identifying restrictions. For the model of the OIS term structure, we follow Dai and Singleton (2000); Hamilton and Wu (2012) and restrict $\mu_u = 0$, $\Sigma_u = I$, $\delta_{u1} \geq 0$, Φ_u^* lower triangular with real eigenvalues and $\phi_{u11}^* \geq \phi_{u22}^*$. For the credit model, we impose the mean of state variables g_t to be equal to 1 under the objective probability to avoid scaling indeterminacy. This implies restrictions on parameters $c_i : c_i \nu_i = 1 - \phi_i^\top \iota$, where ι is a vector of ones. By the same logic, we set $\rho = 1$ in the objective dynamics of the contagion factor d_t .

Because of the large number of parameters, we impose overidentifying restrictions as well. We allow the global credit factor w_{1t} to affect regional factors, but not vice-versa. This restriction affects elements ϕ_i . We assume the volatility factor v_t to be autonomous under both probabilities. Both sets of restrictions translate into identical restrictions under risk-neutral probability because of the functional form of the prices of risk for these factors. Further, we assume that the contagion factor d_t loads only on w_{1t} under the objective probability.

4 Results

4.1 Model selection and fit

Table 4 displays estimated parameters of the OIS term structure. The model is standard and parameter values are in line with the literature. It is difficult to interpret the model because there are multiple equivalent rotations of the factors. The key for this paper is that, as indicated in Table 8, the pricing errors are reasonably small. Therefore, we can use the model for discounting cashflows using the risk-neutral valuation method.

We estimate two versions of the credit model: with and without contagion. We present estimated parameters in Tables 5 and 6, respectively. There are common traits to parameter estimates regardless of the model. The factors g_t exhibit near unit-root dynamics under risk-neutral probability, a common trait of affine models. Under actual probability, only the $g_{1t} = w_{1t}$ reflecting the global credit factor is highly persistent.

Further, the peripheral countries in our sample have a much larger weight on the global credit factor than the core ones. France has a smaller loading on the core credit factor than Belgium (Germany's weight is zero by assumption). Portugal's weight on the peripheral factor is much larger than those of Ireland and Italy.

We find that the expected depreciation rate loads on v_t only under both probabilities, ditto for shocks. The FX jump magnitude is less than 1%, on average, under actual probability. This is consistent with visually mild observed movements in Figure 1B. Under risk-neutral probability, it is 6% implying a huge risk premium associated with currency devaluation upon a sovereign credit event. This number is connected to the first term in equation (3) although they are not identical due to convexity of the exponential function.

We evaluate whether a larger model with contagion is supported by the evidence. The parameters that are contagion-specific, δ_d^{*k} , δ_d , and ρ^* are statistically significant. The question is whether the extra degrees of freedom associated with a larger model are justified from the statistical and economic perspectives. While we find some credence to the contagion mechanism in our sample, the improvement in the model's fit does not justify the associated increase in statistical uncertainty.

Specifically, Table 7 reports the distributions of the likelihoods of both models, and the associated BIC's (negative of the likelihood plus penalty for the number of parameters). Both statistics indicate that the difference between the two models is insignificant. Table 8 reports various measures of pricing errors for the model without contagion. The same metrics for the model with contagion are similar, and, therefore, not reported for brevity. Thus, we discuss only the model without contagion in the sequel.

Continuing with Table 8 we see that the overall fit is good with RMSE ranging from 2 to 17 basis points. To visualize the fit, we plot the time series of the observed and fitted spreads in Figure 5. The overall high quality of fit is evident.

4.2 Hazard rates and quanto spreads

Figure 6 displays filtered latent states. Recall that one of the identifying restrictions is that the mean of the factors g_t are set to 1. This explains similarity in scale. The time-series patterns are quite different. All credit factors w_t exhibit large movements in 2012, around the height of the sovereign debt crisis. This period was associated with downgrades of individual sovereigns, the European Financial Stability Facility, and political instability in Greece. The crisis reached its peak when Greece officially defaulted in March 2012.

The sharp drop in all credit factors shortly thereafter is associated with the famous speech by Mario Draghi in July 2012, vowing to do “whatever it takes” to save the EUR. While all the credit factors persistently decrease thereafter, the FX volatility factor exhibits more pronounced spikes in 2015. Thus, the FX volatility factor displays two different periods of elevated volatility, one that is associated with high and one with low sovereign risk.

The peripheral factor w_{3t} , associated with Italy, Ireland, and Portugal, also exhibits substantial variation prior to 2012, and to a smaller degree, for the rest of the sample period. The core factor w_{2t} , which corresponds to Belgium and France, starts to pick shortly after the marked increase in w_{3t} , but much earlier than the pronounced increase of the common factor w_{1t} . Interestingly, $w_{3,t}$ starts to pick up again at the end of the sample period in 2016, while the other two credit factors continue their steady downward trend.

Various combinations of these factors deliver risk-neutral hazard rates for the respective countries via equation (7). Figure 7A displays these hazard rates. Given the common variation in credit factors during 2012, it is not surprising to see elevated default probabilities during that period irrespective of the country. The largest one-period default probability is 1.5%. Ireland, Italy, and Portugal have a clearly distinct patterns of hazard rates in the post-2012 period, a manifestation of the exposure to the factor w_{3t} . Overall, Portugal is the most risky across all countries. Germany does not load on regional components at all, and its loading on the global component is the lowest. Because of that, Germany not only has the lowest default risk, but also its dynamic properties are distinct from those of other countries.

These differences in dynamics manifest themselves in the differences between the average quanto curves, displayed in Figure 8. We see that the visible differences between the countries that we observed at the outset of our analysis get washed away at longer horizons. Thus, in the long run, there is a steady state conditional (upon default) expected depreciation that is no longer driven by regional differences in default probabilities. However, the difference between Germany and the rest of the countries is permanent because of the differential exposure to the regional factors. Contrast these patterns with those in Figure 1A. As we have normalized each curve by the one-year quanto spread, the lines diverge. The steepness of the term structures is informative about the speed of convergence to the steady state.

While we cannot characterize default risk premiums for individual countries, we can do so for the overall credit risk. Figure 7B displays aggregate hazard objective and risk-neutral hazard rates Λ_t and Λ_t^* , respectively computed from jump intensities via $\Lambda_t = 1 - \exp(-\lambda_t)$ (and the same with asterisks). We characterize the corresponding risk premium via $\log \lambda_t^*/\lambda_t$, displayed in Figure 7C. On average, this number is 2.2, drifting upwards towards the end of the sample. As a benchmark, [Driessen \(2005\)](#) assumes a constant default premium in the context of corporate debt and estimates it to be 2.3, but in levels rather than logs. Combining CDS-implied default intensities with Moody’s KMV expected default frequencies, [Berndt, Douglas, Duffie, Ferguson, and Schranz \(2008\)](#) also find average default premia around 2 for a sample of 93 firms in three industries: broadcasting and entertainment, healthcare, and oil and gas. The ratios of risk-neutral to physical default intensities exhibit, however, substantial time variation, and go up as high as 10. Thus, our estimate is very large, reflecting an extremely low true probability of a default. [Bai, Collin-Dufresne, Goldstein, and Helwege \(2015\)](#); [Gouriéroux, Monfort, and Renne \(2014\)](#) emphasize that if a model of default intensity is missing the contagion effect, then the ratio of intensities might be overstating the actual premium for credit risk. In our case, a version of the model with contagion generates an average premium of 2.3 (in logs also) and substantively similar dynamics.

4.3 Expected devaluation and relative quanto spreads

Several authors, e.g., [Du and Schreger \(2016\)](#); [Mano \(2013\)](#), use observed relative quanto spreads to measure anticipated currency devaluation in case of a credit event (EUR in our case). Equation (3) shows that the relative spread consists of two components, and risk-neutral expected depreciation conditional on default is one of them. Our model allows us to gauge how close this term is to the full relative spread.

One might want to consider θ^* – a risk-neutral mean jump in EUR in case of a credit event as a measure of the first term in equation (3). However, that term is expressed in currency levels, while θ^* pertains to logs. The distinction between the expectation of the FX rate and the exponential of the mean of its log could be important here because it is driven by time-variation in the depreciation rate’s variance and jump rate. We rely on the same techniques that we have used for the valuation of CDS contracts and evaluate the first term in equation (3) in the estimated model. Next, we compare it to the observed relative quanto spread.

To develop intuition, we start with the case that ignores the timing of default, and compute both the risk-neutral expectation, $E_t^*[1 - S_T/S_t]$, and its objective counterpart, $E_t[1 - S_T/S_t]$. We see in the first row of Figure 9 that the objective expectations indicate expected EUR devaluation, and the average term structure of such expectations is upward sloping. The risk-neutral expectation shows that the USD is expected to devalue, on average, at horizons up to 4 years.

To understand this result, consider a case of a one-period expectation. Covered Interest Parity implies $E_t^*[1 - S_{t+T}/S_t] = 1 - \exp(y_{t,T} - \hat{y}_{t,T})$ with $\hat{y}_{t,T}$ denoting the Euro benchmark yield. Thus, the expectation is negative whenever $y_{t,T} > \hat{y}_{t,T}$.

The second row of Figure 9 conditions on the timing τ of any credit event rather than the one in a specific country. In this case, the arrival rate is controlled by λ_t , and we can compute both the risk-neutral expectation and its objective counterpart. Default risk has almost no impact over one period, so the plot for $T = 1$ week is similar to that for the no-default case. The level of the expectations adjusts downwards at longer maturities, indicating risk-neutral expected USD depreciation conditional on default. This happens because early termination leads to a loss in the expected USD appreciation. The Figure suggests that the corresponding risk premium for this event is substantial.

Indeed we can compute excess returns associated with the two scenarios: $(T - t)^{-1} \log[E_t S_T / E_t^* S_T]$ (no default) and $(T - t)^{-1} \log[E_t S_{t \wedge T} / E_t^* S_{t \wedge T}]$ (default). The difference between the two reflects the compensation for the currency loss in case of default. The third row of Figure 9 displays these differences for $T = 1$ week and 5 years, and the average term-structure. The premium is about the same, 0.06% per week, at short and long horizons. It peaks at 2 years at a substantive 0.45% per week.

Credit events of individual countries arrive at a frequency lower than λ_t^* , by construction. So, we anticipate the risk adjustment to be smaller and, as a result, individual expected depreciation rates to be somewhere in between the the risk-neutral expectations reviewed in Figure 9. Indeed, consistent with this intuition, risk-neutral expectations are occasionally negative, but are higher for individual countries. See Figure 10.

The longer term average risk-neutral expectations in Figure 10 are higher for core countries with the highest one for Germany. Thus, despite higher default probability in periphery countries, the expected currency devaluation is more modest. This result reflects relatively smaller economic performance of this group of countries.

This Figure also shows observed relative quanto spreads of matching maturities. We see that the two objects could be substantively different to each other although the averages appear to be similar at longer horizons. This is confirmed in the last panel of each row, which plot average term structures. These differences suggest that the covariance term in (3) is negative and relatively large. Even at long horizons it contributes between 40% and 60% of the quanto spread, with the exception of Germany.

5 Conclusion

We propose an affine no-arbitrage model of joint dynamics of sovereign CDS quanto spreads in the Eurozone and the USD/EUR exchange rate. Default probabilities vary substantially in the cross-section before converging in the long-run. This feature makes one and the

same exchange rate have effectively different risk-neutral distributions from the perspective of a given country. That interaction between default and the exchange rate commands a substantial risk premium.

References

- Ait-Sahalia, Yacine, Roger J.A. Laeven, and Liora Pelizzon, 2014, Mutual excitation in eurozone sovereign cds, *Journal of Econometrics* 183, 151–167.
- Ang, Andrew, and Francis A. Longstaff, 2013, Systemic sovereign credit risk: Lessons from the u.s. and europe, *Journal of Monetary Economics* 60, 493–510.
- Augustin, Patrick, 2014, Sovereign credit default swap premia, *Journal of Investment Management* 12, 65–102.
- , Valeri Sokolovski, Marti G. Subrahmanyam, and Davide Tomio, 2016, Why do investors buy sovereign default insurance?, *Working Paper New York University, Stern School of Business*.
- Azizpour, Shahriar, Kay Giesecke, and Gustavo Schwenkler, 2017, Exploring the sources of default clustering, *Journal of Financial Economics* Forthcoming.
- Backus, David K., Silverio Foresi, and Chris I. Telmer, 2001, Affine term structure models and the forward premium anomaly, *The Journal of Finance* 56, 279–304.
- Bai, Jennie, Pierre Collin-Dufresne, Robert Goldstein, and Jean Helwege, 2015, On bounding credit event risk premia, *Review of Financial Studies* 28, 2608–2642.
- Benzoni, Luca, Pierre Collin-Dufresne, Robert S. Goldstein, and Jean Helwege, 2015, Modeling credit contagion via the updating of fragile beliefs, *Review of Financial Studies* 28, 1960–2008.
- Berndt, Antje, Rohan Douglas, Darrell Duffie, Mark Ferguson, and David Schranz, 2008, Measuring default risk premia from default swap rates and edfs, *Working Paper*.
- Brigo, Damiano, Nicola Pede, and Andrea Petrelli, 2016, Multi currency credit default swaps: Quanto effects and fx devaluation jumps, *Working Paper*.
- Buraschi, Andrea, Emrah Sener, and Murat Mengueterk, 2014, The geography of funding markets and limits to arbitrage, *Review of Financial Studies* 28, 1103–1152.
- Carr, Peter, and Liuren Wu, 2007, Theory and evidence on the dynamic interactions between sovereign credit default swaps and currency options, *Journal of Banking & Finance* 31, 2383–2403.
- Corte, Pasquale Della, Lucio Sarno, Maik Schmeling, and Christian Wagner, 2016, Exchange rates and sovereign risk, *Working Paper*.
- Dai, Qiang, and Kenneth J. Singleton, 2000, Specification analysis of affine term structure models, *The Journal of Finance* 55, 1943–1978.
- Das, Sanjiv, Darrell Duffie, Nikunj Kapadia, and Leandro Saita, 2007, Common failings: How corporate defaults are correlated, *The Journal of Finance* 62, 93–117.

- DeSantis, Roberto A., 2015, A measure of redenomination risk, *ECB Working Paper 1785*.
- Doshi, Hitesh, Jan Ericsson, Kris Jacobs, and Stuart M. Turnbull, 2013, Pricing credit default swaps with observable covariates, *Review of Financial Studies* 26, 2049–2094.
- Doshi, Hitesh, Kris Jacobs, and Carlos Zurita, 2017, Economic and financial determinants of credit risk premiums in the sovereign cds market, *Review of Asset Pricing Studies* 1, 43–80.
- Driessen, Joost, 2005, Is default event risk priced in corporate bonds?, *The Review of Financial Studies* 18, 165–195.
- Du, Wenxin, and Jesse Schreger, 2016, Local currency sovereign risk, *The Journal of Finance* 71, 1027–1070.
- Duffee, Gregory R., 1999, Estimating the price of default risk, *Review of Financial Studies* 12, 197–226.
- , 2002, Term premia and interest rate forecasts in affine models, *The Journal of Finance* 57, 405–443.
- Duffie, Darrell, Lasse H. Pedersen, and Kenneth J. Singleton, 2003, Modeling sovereign yield spreads: A case study of russian debt, *The Journal of Finance* 58, 119–159.
- Duffie, Darrell, and Kenneth J. Singleton, 2003, *Credit Risk: Pricing, Measurement and Management* (Princeton University Press).
- Ehlers, Philippe, and Philipp Schoenbucher, 2004, The influence of fx risk on credit spreads, *Working Paper*.
- Feldhutter, Peter, and David Lando, 2008, Decomposing swap spreads, *Journal of Financial Economics* 88, 375–405.
- Giglio, Stefano, 2014, Credit default swap spreads and systemic financial risk, *Working Paper*.
- Gourieroux, Christian, and Joann Jasiak, 2006, Autoregressive gamma processes, *Journal of Forecasting* 25, 129–152.
- Gouriéroux, Christian, Alain Monfort, and Jean-Paul Renne, 2014, Pricing default events: Surprise, exogeneity and contagion, *Journal of Econometrics* 182, 397–411.
- Hamilton, James, and Cynthia Wu, 2012, Identification and estimation of Gaussian affine term structure models, *JE* 168, 315–331.
- Herbst, E., and F. Schorfheide, 2016, *Bayesian Estimation of DSGE Models* (Princeton University Press).
- Hoerdahl, Peter, and Oreste Tristani, 2012, The term structure of euro area sovereign bond yields, *BIS Working Paper*.

- Hull, John, and Alan White, 2013, Libor vs. ois: The derivatives discounting dilemma, *Journal Of Investment Management* 11, 14–27.
- Le, Anh, Kenneth J. Singleton, and Qiang Dai, 2010, Discrete-time affine term structure models with generalized market prices of risk, *Review of Financial Studies* 23, 2184–2227.
- Longstaff, Francis A., Jun Pan, Lasse H. Pedersen, and Kenneth J. Singleton, 2011, How sovereign is sovereign credit risk?, *American Economic Journal: Macroeconomics* 3, 75–103.
- Mano, Rui, 2013, Exchange rates upon sovereign default, *Working Paper*.
- Monfort, Alain, Fulvio Pegoraro, Jean-Paul Renne, and Guillaume Rousselet, 2014, Staying at zero with affine processes: a new dynamic term structure model, *Journal of Econometrics*.
- , 2017, Affine modeling of credit risk, credit event and contagion, *Working Paper*.
- Monfort, Alain, and Jean-Paul Renne, 2013, Decomposing euro-area sovereign spreads: Credit and liquidity risks, *Review of Finance* 18, 2103–2151.
- Na, S., S. Schmitt-Grohé, M. Uribe, and V. Yue, 2017, The twin ds: Optimal default and devaluation, *Columbia University Working Paper*.
- Pan, Jun, and Kenneth J. Singleton, 2008, Default and recovery implicit in the term structure of sovereign cds spreads, *The Journal of Finance* 63, 2345–2384.
- Qiu, Jiaping, and Fan Yu, 2012, Endogenous liquidity in credit derivatives, *Journal of Financial Economics* 103, 611–631.
- Reinhart, Carmen M., 2002, Default, currency crises, and sovereign credit ratings, *World Bank Economic Review* 16, 151–170.
- Siriwardane, E., 2014, Concentrated capital losses and the pricing of corporate credit risk, *Working Paper*.
- Zhang, Frank Xiaoling, 2008, Market expectations and default risk premium in credit default swap prices: A study of argentine default, *Journal of Fixed Income* 18, 37–55.

A Term structure of quanto spreads in the iid case

We show that the term structure of quanto spreads is flat when hazard and depreciation rates are iid, and the risk-free interest rate is constant. To achieve analytical tractability, we replace CDS premiums with credit spreads.

We consider a bond with no coupons that pays \$1 at maturity if there is no default. Default may take place any time prior to maturity. Thus, the bond's random payoff is

$$\mathcal{X}_\tau^n = \mathcal{I}(\tau > t + n) + (1 - L_\tau) \cdot \mathcal{I}(\tau \leq t + n),$$

where L_τ is loss given default. We consider the bond's value, $V_{t,T}^\epsilon$ for the case known as recovery of market value (RMV), $1 - L_\tau = (1 - L) \cdot V_{\tau,T}^\epsilon$.

Under the RMV assumption, the bond price is given by

$$\begin{aligned} V_{t,T}^\epsilon &= E_t^* \left(B_{t,t} \frac{S_{t+1}}{S_t} [\mathcal{I}(\tau > t + 1) + (1 - L) \cdot \mathcal{I}(t < \tau \leq t + 1)] \cdot V_{t+1,T}^\epsilon \right) \\ &= E_t^* \left(B_{t,t} \frac{S_{t+1}}{S_t} [(1 - H_{t+1}^*) + (1 - L)H_{t+1}^*] \cdot V_{t+1,T}^\epsilon \right) \\ &= E_t^* \left(B_{t,t} \frac{S_{t+1}}{S_t} [1 - LH_{t+1}^*] \cdot V_{t+1,T}^\epsilon \right) = E_t^* e^{-\sum_{j=1}^n (r_{t+j-1} + \tilde{h}_{t+j}^* - \Delta s_{t+j})} \\ &= \left[e^{-r} E_t^* e^{\Delta s_{t+1} - \tilde{h}_{t+1}^*} \right]^{T-t}, \end{aligned}$$

where $\tilde{h}_t^* = -\log(1 - LH_t^*) \approx Lh_t^*$ (based on the first-order Taylor approximation), and the last line is implied by the assumptions of this section.

Therefore, the credit spread on this bond is $c_{t,T}^\epsilon = -\log E_t^* e^{\Delta s_{t+1} - \tilde{h}_{t+1}^*}$. Similarly, $c_{t,T}^\$ = -\log E_t^* e^{-\tilde{h}_{t+1}^*}$. Thus, the term spread $[c_{t,T_2}^\$ - c_{t,T_2}^\epsilon] - [c_{t,T_1}^\$ - c_{t,T_1}^\epsilon] = 0$.

B The affine pricing kernel

A multiperiod pricing kernel is a product of one-period ones: $M_{t,t+n} = M_{t,t+1} \cdot M_{t+1,t+2} \cdot \dots \cdot M_{t+n-1,t+n}$. In this section, we specify the pricing kernel and show how the associated prices of risk modify the distribution of state x_t .

The (log) pricing kernel is:

$$m_{t,t+1} = -r_t - k_t(-\gamma_{x,t}; \varepsilon_{x,t+1}) - k_t(-1; z_{m,t+1}) - \gamma_{x,t}^\top \varepsilon_{x,t+1} - z_{m,t+1},$$

where $k_t(s; e_{t+1}) = \log E_t e^{s \cdot e_{t+1}}$ is the cumulant-generating function (cgf), and $z_{m,t+1}$ is a jump process with intensity λ_{t+1} . The behavior of risk premiums is determined by $\gamma_{x,t}$ and the jump magnitude $z_{m,t+1}|j_{t+1}$.

In the case of factor u_t , the risk premium is $\gamma_{u,t} = \Sigma_u^{-1}(\bar{\gamma}_u + \delta_u u_t)$ implying

$$\Phi_u^* = \Phi_u - \delta_u, \quad \mu_u^* = \mu_u - \bar{\gamma}_u$$

with $k_t(-\gamma_{u,t}; \varepsilon_{u,t+1}) = \gamma_{u,t}^\top \gamma_{u,t}/2$. In the case of factor g_t , the risk premium is $\gamma_{g,t} = \bar{\gamma}_g$ implying

$$\phi_{ij}^* = \phi_{ij}(1 - \bar{\gamma}_{gi}c_i)^{-2}, \quad c_i^* = c_i(1 - \bar{\gamma}_{gi}c_i)^{-1}$$

with

$$k_t(-\gamma_{g,t}; \eta_{g,t+1}) = - \sum_{i=1}^{M_g} \left(\nu_i [\log(1 + \gamma_{gi,t}) - \gamma_{gi,t}c_i] + \gamma_{gi,t}[(1 + \gamma_{gi,t}c_i)^{-1} - 1] \phi_i^\top g_t \right).$$

See [Le, Singleton, and Dai \(2010\)](#).

In the case of jumps, the risk premium is

$$z_{m,t+1}|j_{t+1} = \sum_k -(h_{t+1}^k - h_{t+1}^{*k}) - j_{t+1}^k \log h_{t+1}^{*k}/h_{t+1}^k + j_{t+1}^k \log \theta^*/\theta + (\theta^{*-1} - \theta^{-1}) \cdot z_{s,t+1}|j_{t+1}^k$$

implying risk-neutral jump $z_{s,t+1}$ with Poisson arrival rate of $\lambda_{t+1}^* = \sum_k h_{t+1}^{*k}$ and magnitude $z_{s,t+1}|j_{t+1} \sim \text{Gamma}(j_{t+1}, \theta^*)$. The corresponding cgf is $k_t(-1; z_{m,t+1}) = 0$.

Indeed, a Poisson mixture of gammas distribution implies arbitrary form of risk premiums without violating no-arbitrage conditions. To see this, first observe that

$$p_{t+1}(z_s|j) = \frac{e^{-\lambda_{t+1}} \lambda_{t+1}^j}{j!} \frac{1}{\Gamma(j)\theta^j} z_s^{j-1} e^{-z_s/\theta}.$$

Second, assume that risk-neutral distribution features an arbitrary arrival rate λ_{t+1}^* and jump size mean θ^* (this does not have to be a constant). Then

$$p_{t+1}^*(z_s|j) = \frac{e^{-\lambda_{t+1}^*} \lambda_{t+1}^{*j}}{j!} \frac{1}{\Gamma(j)\theta^{*j}} z_s^{j-1} e^{-z_s/\theta^*}.$$

We characterize the ratio $p_{t+1}^*(z)/p_{t+1}(z)$ via the moment-generating function (mgf) of its log. First, we compute expectation with respect to jump-size distribution

$$\begin{aligned} \tilde{h}_{t+1}(s; \log p_{t+1}^*(z_s)/p_{t+1}(z_s)) &= E_{t+1} e^{s \log p_{t+1}^*(z_s)/p_{t+1}(z_s)} \\ &= \sum_{j=0}^{\infty} \frac{e^{-\lambda_{t+1}} \lambda_{t+1}^j}{j!} e^{(\lambda_{t+1} - \lambda_{t+1}^*) + j \log \lambda_{t+1}^*/\lambda_{t+1} - j \log \theta^*/\theta} (1 - s\theta(\theta^{-1} - \theta^{*-1}))^{-j}. \end{aligned}$$

This functional form of the mgf reflects a Poisson mixture with intensity λ_{t+1} and magnitude $z_m|j = (\lambda_{t+1} - \lambda_{t+1}^*) + j \log \lambda_{t+1}^*/\lambda_{t+1} - j \log \theta^*/\theta - (\theta^{*-1} - \theta^{-1}) \cdot z_s|j$. The expression could be simplified further:

$$\begin{aligned} \tilde{h}_{t+1}(s; \log p_{t+1}^*(z_s)/p_{t+1}(z_s)) &= \sum_{j=0}^{\infty} \frac{e^{-\lambda_{t+1}}}{j!} [\lambda_{t+1}^* \theta/\theta^* (1 - s\theta(\theta^{-1} - \theta^{*-1}))^{-1}]^j \\ &= e^{\lambda_{t+1}^* (\theta/\theta^* (1 - s\theta(\theta^{-1} - \theta^{*-1}))^{-1} - 1)}. \end{aligned}$$

Second, we obtain the mgf by computing expectation with respect to the distribution of jump intensity:

$$h_t(s; \log p_{t+1}^*(z_s)/p_{t+1}(z_s)) = E_t \tilde{h}_{t+1}(s; \log p_{t+1}^*(z_s)/p_{t+1}(z_s)) \equiv E_t e^{\lambda_{t+1}^* f(s, \theta, \theta^*)}.$$

The cgf is

$$k_t(s; \log p_{t+1}^*(z_s)/p_{t+1}(z_s)) = \log E_t e^{\lambda_{t+1}^* f(s, \theta, \theta^*)} = k_t(f(s, \theta, \theta^*), \lambda_{t+1}^*).$$

Note that $k_t(-1; z_{m,t+1})$ corresponds to $k_t(1; \log p_{t+1}^*(z_s)/p_{t+1}(z_s))$, and $f(1, \theta, \theta^*) = 0$, so $k_t(-1; z_{m,t+1}) = 0$.

C Details of bond valuation

To derive closed-form solutions for the term structure of interest rates, we conjecture that zero-coupon bond prices $Q_{t,T}$ are exponentially affine in the state vector u_t

$$q_{t,T} = \log Q_{t,T} = -\tilde{A}_{T-t} - \tilde{B}_{T-t}^\top u_t.$$

Because the interest rate is an affine function of the state, $r_t = \bar{r} + \delta_r^\top u_t$, log-bond prices are fully characterized by the cumulant-generating function of u_t . The law of iterated expectations implies that $Q_{t,T}$ satisfies the recursion

$$Q_{t,T} = e^{-r_t} E_t^* Q_{t+1,T-1},$$

It can be shown that for all $n = 0, 1, \dots, T-t$, the scalar $\tilde{A}_n$ and components of the column vector $\tilde{B}_n$ are given by

$$\begin{aligned} \tilde{A}_n &= \tilde{A}_{n-1} + \bar{r} + \tilde{B}_{n-1}^\top \mu_u - \frac{1}{2} \tilde{B}_{n-1}^\top \Sigma_u \Sigma_u^\top \tilde{B}_{n-1} \\ \tilde{B}_n^\top &= \delta_r^\top + \tilde{B}_{n-1}^\top \Phi_u, \end{aligned}$$

with initial conditions $\tilde{A}_0 = 0$ and $\tilde{B}_0 = 0$.

Then the yields are

$$y_{t,T} = -(T-t)^{-1} \log Q_{t,T} = A_{T-t} + B_{T-t}^\top u_t$$

with $A_{T-t} = -(T-t)^{-1} \tilde{A}_{T-t}$ and $B_{T-t} = -(T-t)^{-1} \tilde{B}_{T-t}$.

D Details of CDS valuation

Actual implementation of CDS valuation extends equation (11) by accounting for accrual payments:

$$C_{t,T}^{\epsilon} = L \cdot \frac{\sum_{j=1}^{T-t} (\tilde{\Psi}_{j,t} - \Psi_{j,t})}{\sum_{j=1}^{(T-t)/\Delta} \Psi_{j\Delta,t} + \sum_{j=1}^{T-t} \left(\frac{j}{\Delta} - \lfloor \frac{j}{\Delta} \rfloor \right) (\tilde{\Psi}_{j,t} - \Psi_{j,t})},$$

where the floor function $\lfloor \cdot \rfloor$ rounds to the nearest lower integer. The law of iterated expectations implies that $\tilde{\Psi}_{j,t}$ and $\Psi_{j,t}$ satisfy the recursions

$$\tilde{\Psi}_{j,t} = E_t^* \left[B_{t,t} (1 - H_{t+1}) \frac{S_{t+1}}{S_t} \tilde{\Psi}_{j-1,t+1} \right], \quad \Psi_{j,t} = E_t^* \left[B_{t,t} (1 - H_{t+1}) \frac{S_{t+1}}{S_t} \Psi_{j-1,t+1} \right],$$

starting at $j = 1$ for $\tilde{\Psi}_{j,t}$ and at $j = 0$ for $\Psi_{j,t}$. To evaluate the expressions for $\tilde{\Psi}$ and Ψ , we conjecture that these expressions are exponentially affine functions of the state vector x_t . See equation (14).

We define the column vectors $\tilde{B}_j = [\tilde{B}_{u,j}^\top, \tilde{B}_{g,j}^\top, \tilde{B}_{d,j}^\top]^\top$, with $u = 1, 2, \dots, M_u$, $g = 1, 2, \dots, M_g$, and $d = 1, 2, \dots, M_d$. Next, the column vectors of ones with length M_u , M_g , and M_d are denoted by $\mathbb{1}_{M_u}$, $\mathbb{1}_{M_g}$, and $\mathbb{1}_{M_d}$, respectively. Define the matrices $\Delta_w^* = [\delta_w^{*1}, \delta_w^{*2}, \dots, \delta_w^{*M_c}]$ and $\Delta_d^* = [\delta_d^{*1}, \delta_d^{*2}, \dots, \delta_d^{*M_c}]$. Finally, we subdivide the vector δ_s^* into sub-matrices $\delta_s^{*\top} = [\delta_{su}^{*\top}, \delta_{sg}^{*\top}, \delta_{sd}^{*\top}]$, for $u = 1, 2, \dots, M_u$, $g = 1, 2, \dots, M_g$, and $d = 1, 2, \dots, M_d$. It can be shown that for each country $k = 1, 2, \dots, M_c$ and for all

$j = 1, 2, \dots, T - t$, the scalar $\tilde{A}_j$ and components of the column vectors $\tilde{B}_j$ are given by

$$\begin{aligned}
\tilde{A}_j &= \tilde{A}_{j-1} - \bar{r} - \bar{h}^{*k} + \bar{s}^* + \frac{1}{2} \bar{v}^* - \bar{h}^* \left[\frac{\theta^*}{1 - \theta^*} \right] + \left(\tilde{B}_{u,j-1} + \delta_{su}^* \right)^\top \mu_u \\
&\quad + \frac{1}{2} \left(\tilde{B}_{u,j-1} + \delta_{su}^* \right)^\top \Sigma_u \Sigma_u^\top \left(\tilde{B}_{u,j-1} + \delta_{su}^* \right) + \left[\frac{\tilde{B}_{d,j-1} \odot \bar{H}^*}{\mathbb{1}_{M_d} - \tilde{B}_{d,j-1} \odot \rho^*} \right]^\top \cdot \mathbb{1}_{M_d} \\
&\quad - \left(\nu \odot \log \left[\mathbb{1}_{M_g} - \left(\tilde{B}_{g,j-1} + \delta_{sg}^* - \delta_w^{*k} - \delta_w^* \odot \left[\frac{\theta^*}{1 - \theta^*} \right] \right. \right. \right. \\
&\quad \left. \left. \left. + \left[\Delta_w^* \left(\frac{\tilde{B}_{d,j-1}}{\mathbb{1}_{M_d} - \tilde{B}_{d,j-1} \odot \rho^*} \right) \right] \right) \odot c^* \right] \right)^\top \cdot \mathbb{1}_{M_g} \\
\tilde{B}_{u,j} &= \Phi_u^\top \left[\tilde{B}_{u,j-1} + \delta_{su}^* \right] - \delta_r \\
\tilde{B}_{g,j} &= \Phi_g^{*\top} \left(\frac{\tilde{B}_{g,j-1} + \delta_{sg}^* - \delta_w^{*k} - \delta_w^* \odot \left[\frac{\theta^*}{1 - \theta^*} \right] + \left[\Delta_w^* \left(\frac{\tilde{B}_{d,j-1}}{\mathbb{1}_{M_d} - \tilde{B}_{d,j-1} \odot \rho^*} \right) \right]}{\mathbb{1}_{M_g} - \left[\tilde{B}_{g,j-1} + \delta_{sg}^* - \delta_w^{*k} - \delta_w^* \odot \left[\frac{\theta^*}{1 - \theta^*} \right] + \left[\Delta_w^* \left(\frac{\tilde{B}_{d,j-1}}{\mathbb{1}_{M_d} - \tilde{B}_{d,j-1} \odot \rho^*} \right) \right] \right] \odot c^*} \right) \\
&\quad + \frac{1}{2} \delta_v^* \\
\tilde{B}_{d,j} &= \Delta_d^{*\top} \left(\frac{\tilde{B}_{d,j-1}}{\mathbb{1}_{M_d} - \tilde{B}_{d,j-1} \odot \rho^*} \right) - \delta_d^{*k} - \delta_d^* \odot \left[\frac{\theta}{1 - \theta} \right],
\end{aligned} \tag{15}$$

where $\odot$ defines the Hadamard product for element-wise multiplication, and where we slightly abuse notation as the division and log operators work element-by-element when applied to vectors or matrices. The recursions for Ψ are identical. It is sufficient to replace $\tilde{A}$ and $\tilde{B}$ by A and B , respectively.

The initial condition for $\tilde{\Psi}$ is given by

$$\tilde{\Psi}_{1,t} = e^{\tilde{A}_1 + \tilde{B}_1^\top x_t}, \tag{16}$$

where the scalar $\tilde{A}_1$ and components of the column vectors $\tilde{B}_1 = \left[\tilde{B}_{u,1}^\top, \tilde{B}_{g,1}^\top, \tilde{B}_{d,1}^\top \right]^\top$ are

given by

$$\begin{aligned}
\tilde{A}_1 &= \bar{s}^* - \bar{r} + \delta_{su}^{*\top} u_u + \frac{1}{2} \delta_{su}^{*\top} \Sigma_u \Sigma_u^\top \delta_{su}^* + \frac{1}{2} \bar{v}^* \\
&\quad - \left[\nu \odot \log \left(\mathbb{1}_{M_g} - \left(\delta_{sg}^* - \delta_w^* \odot \left[\frac{\theta^*}{1 - \theta^*} \right] \right) \odot c^* \right) \right]^\top \cdot \mathbb{1}_{M_g} - \bar{h}^* \left[\frac{\theta^*}{1 - \theta^*} \right] \\
\tilde{B}_{u,1} &= \Phi_u^\top \delta_{su}^* - \delta_r \\
\tilde{B}_{g,1} &= \Phi_g^{*\top} \left(\frac{\delta_{sg}^* - \delta_w^* \odot \left[\frac{\theta^*}{1 - \theta^*} \right]}{\mathbb{1}_{M_g} - \left(\delta_{sg}^* - \delta_w^* \odot \left[\frac{\theta^*}{1 - \theta^*} \right] \right) \odot c^*} \right) + \frac{1}{2} \delta_v^* \\
\tilde{B}_{d,1} &= -\delta_d^* \odot \left[\frac{\theta^*}{1 - \theta^*} \right].
\end{aligned} \tag{17}$$

The initial condition for Ψ is given by

$$\Psi_{0,t} = e^{A_0 + B_0^\top x_t}, \tag{18}$$

where the scalar $A_0 = 0$ and the column vector $B_0 = 0$.

The pricing equation for the USD-CDS spread is obtained in closed form by setting $S_{t+j} = 1$ in all recursions.

E Details of estimation

E.1 State-space representation

State transition equation

We consider two interest rate factors $u_{1,t}, u_{2,t}$, three credit factors $g_{1,t}, g_{2,t}, g_{3,t}$, one volatility factor $g_{4,t} = v_t$, and one contagion factor d_t

$$\underbrace{\begin{bmatrix} u_{1,t+1} \\ u_{2,t+1} \end{bmatrix}}_{u_{t+1}} = \underbrace{\begin{bmatrix} \mu_{u1} \\ \mu_{u2} \end{bmatrix}}_{\mu_u} + \underbrace{\begin{bmatrix} \phi_{u11} & \phi_{u12} \\ \phi_{u21} & \phi_{u22} \end{bmatrix}}_{\Phi_u} \underbrace{\begin{bmatrix} u_{1,t} \\ u_{2,t} \end{bmatrix}}_{u_t} + \underbrace{\begin{bmatrix} \eta_{u1,t+1} \\ \eta_{u2,t+1} \end{bmatrix}}_{\eta_{u,t+1}} \tag{19}$$

$$\underbrace{\begin{bmatrix} g_{1,t+1} \\ g_{2,t+1} \\ g_{3,t+1} \\ g_{4,t+1} \end{bmatrix}}_{g_{t+1}} = \underbrace{\begin{bmatrix} \nu_{g1} c_{g1} \\ \nu_{g2} c_{g2} \\ \nu_{g3} c_{g3} \\ \nu_{g4} c_{g4} \end{bmatrix}}_{\mu_g} + \underbrace{\begin{bmatrix} \phi_{g11} & \phi_{g12} & \phi_{g13} & \phi_{g14} \\ \phi_{g21} & \phi_{g22} & \phi_{g23} & \phi_{g24} \\ \phi_{g31} & \phi_{g32} & \phi_{g33} & \phi_{g34} \\ \phi_{g41} & \phi_{g42} & \phi_{g43} & \phi_{g44} \end{bmatrix}}_{\Phi_g} \underbrace{\begin{bmatrix} g_{1,t} \\ g_{2,t} \\ g_{3,t} \\ g_{4,t} \end{bmatrix}}_{g_t} + \underbrace{\begin{bmatrix} \eta_{g1,t+1} \\ \eta_{g2,t+1} \\ \eta_{g3,t+1} \\ \eta_{g4,t+1} \end{bmatrix}}_{\eta_{g,t+1}} \tag{20}$$

$$d_{t+1} = \mu_d + \underbrace{\delta_{d,g} \left(\underbrace{\mu_g + \Phi_g g_t + \eta_{g,t+1}}_{g_{t+1}} \right)}_{g_{t+1}} + \Phi_d d_t + \eta_{d,t+1} \tag{21}$$

where $\eta_{u,t} \sim N(0, \Sigma_u \Sigma_u')$, $\eta_{g,t}, \eta_{d,t}, \eta_{\lambda,t}$ represent a martingale difference sequence (mean zero shock). In vector notation, the joint dynamics are

$$\underbrace{\begin{bmatrix} u_{t+1} \\ g_{t+1} \\ d_{t+1} \end{bmatrix}}_{x_{t+1}} = \underbrace{\begin{bmatrix} \mu_u \\ \mu_g \\ \mu_d + \delta_{d,g}\mu_g \end{bmatrix}}_{\mu_x} + \underbrace{\begin{bmatrix} \Phi_u & 0 & 0 \\ 0 & \Phi_g & 0 \\ 0 & \delta_{d,g}\Phi_g & \Phi_d \end{bmatrix}}_{\Phi_x} \underbrace{\begin{bmatrix} u_t \\ g_t \\ d_t \end{bmatrix}}_{x_t} + \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \delta_{d,g} & 1 \end{bmatrix}}_{\Omega_x} \underbrace{\begin{bmatrix} \eta_{u,t+1} \\ \eta_{g,t+1} \\ \eta_{d,t+1} \end{bmatrix}}_{\eta_{x,t+1}}. \quad (22)$$

Here, we are assuming that we observe the sequence $u_{1:T}$. Note that while $\mu_d, \delta_{d,g}, \Phi_d$, which govern the objective dynamics, are estimated freely, we impose

$$\mu_d^* = \sum_{k=1}^{M_c} \bar{h}^{*,k}, \quad \delta_{d,g}^* = \left[\sum_{k=1}^{M_c} \delta_{h,g1}^{*,k} \quad \sum_{k=1}^{M_c} \delta_{h,g2}^{*,k} \quad \sum_{k=1}^{M_c} \delta_{h,g3}^{*,k} \quad 0 \right], \quad \Phi_d^* = \sum_{k=1}^{M_c} \delta_{h,d}^{*,k}.$$

the above restriction in the risk neutral dynamics.

Measurement equations

There are two forms of measurement equations. Denote observables in the first measurement equation and the second measurement equation by $y_{1,t}$ and $y_{2,t}$, respectively. Define $y_t = \{y_{1,t}, y_{2,t}\}$ and $Y_{1:t-1} = \{y_1, \dots, y_{t-1}\}$.

The first measurement equation consists of quanto spreads of six different maturities for each country k

$$qs_t^k = \left\{ qs_{t,1y}^k, qs_{t,3y}^k, qs_{t,5y}^k, qs_{t,7y}^k, qs_{t,10y}^k, qs_{t,15y}^k \right\}^\top, \quad (23)$$

and the log depreciation USD/EUR rate. To ease exposition, define

$$A_{1:T}^k = \{A_1^k, \dots, A_T^k\}, \quad B_{1:T}^k = \{B_1^k, \dots, B_T^k\}, \quad C_{1:T}^k = \{C_1^k, \dots, C_T^k\} \quad (24)$$

and $\tilde{A}_{1:T}^k, \tilde{B}_{1:T}^k, \tilde{C}_{1:T}^k$ are defined similarly. The model-implied quanto spread is nonlinear function of the solution coefficients and current and lagged states, which we express as

$$qs_{t,T}^k = \Xi(A_{1:T}^k, B_{1:T}^k, C_{1:T}^k, \tilde{A}_{1:T}^k, \tilde{B}_{1:T}^k, \tilde{C}_{1:T}^k, x_t). \quad (25)$$

Put together, the first measurement equation becomes

$$y_{1,t} = \begin{bmatrix} qs_t^1 \\ \vdots \\ qs_t^k \\ \vdots \\ qs_t^{M_c} \\ \Delta s_t \end{bmatrix} = \begin{bmatrix} \Xi(A_{1:15y}^1, B_{1:15y}^1, C_{1:15y}^1, \tilde{A}_{1:15y}^1, \tilde{B}_{1:15y}^1, \tilde{C}_{1:15y}^1, x_t) \\ \vdots \\ \Xi(A_{1:15y}^k, B_{1:15y}^k, C_{1:15y}^k, \tilde{A}_{1:15y}^k, \tilde{B}_{1:15y}^k, \tilde{C}_{1:15y}^k, x_t) \\ \vdots \\ \Xi(A_{1:15y}^{M_c}, B_{1:15y}^{M_c}, C_{1:15y}^{M_c}, \tilde{A}_{1:15y}^{M_c}, \tilde{B}_{1:15y}^{M_c}, \tilde{C}_{1:15y}^{M_c}, x_t) \\ \bar{s} + \delta_s^\top x_t + \sqrt{v_t} \varepsilon_{s,t} - z_t \end{bmatrix}. \quad (26)$$

The second measurement equation consists of credit events for each country $e_{k,t}$

$$y_{2,t} = \left\{ e_{1,t}, \dots, e_{M_c,t} \right\}. \quad (27)$$

Instead of providing its measurement equation form, we directly express the likelihood function below.

E.2 Implementation

Likelihood function

We exploit the conditional independence between $y_{1,t}$ and $y_{2,t}$. We express

$$\begin{aligned} P(y_{1,t}, y_{2,t} | Y_{1:t-1}, \Theta) &= \int P(y_{1,t}, y_{2,t} | x_t, Y_{1:t-1}, \Theta) P(x_t | x_{t-1}, Y_{1:t-1}, \Theta) P(x_{t-1} | Y_{1:t-1}, \Theta) dx_{t-1} \\ &= \int \underbrace{P(y_{2,t} | x_t, Y_{1:t-1}, \Theta)}_{(A)} \underbrace{P(y_{1,t} | x_t, Y_{1:t-1}, \Theta)}_{(B)} \underbrace{P(x_t | x_{t-1}, Y_{1:t-1}, \Theta)}_{(C)} P(x_t | Y_{1:t-1}, \Theta) dx_{t-1}, \end{aligned} \quad (28)$$

where (C) can be deduced from (22).

The likelihood function corresponding to (A) in (28) can be written as

$$P(y_{1,t} | x_t, Y_{1:t-1}, \Theta) = (2\pi)^{-n_1/2} |V_1|^{-1/2} \exp \left\{ -\frac{1}{2} (y_{1,t} - \hat{y}_{1,t})^\top V_1^{-1} (y_{1,t} - \hat{y}_{1,t}) \right\} \quad (29)$$

where n_1 is the dimensionality of the vector space, V_1 is a measurement error variance matrix, and $\hat{y}_{1,t}$ is from (26).

The likelihood function corresponding to (B) can be expressed as

$$P(y_{2,t} | x_t, Y_{1:t-1}, \Theta) = \exp(-M_c \lambda_t) \prod_{i=k}^{M_c} \left\{ e_{k,t} \lambda_t + (1 - e_{k,t}) \right\} \quad (30)$$

following (Das, Duffie, Kapadia, and Saita, 2007).

Bayesian inference

For convenience, parameters associated with factors, hazard rates, exchange rate, and defaults are collected in Θ_g , Θ_h , Θ_s , Θ_d respectively.

$$\begin{aligned}\Theta_g &= \left\{ \{\phi_{g11}^*, \phi_{g21}^*, \phi_{g22}^*, \phi_{g31}^*, \phi_{g33}^*, \phi_{g44}^*\}, \{\phi_{g11}, \phi_{g21}, \phi_{g22}, \phi_{g31}, \phi_{g33}, \phi_{g44}\}, \dots \right. \\ &\quad \left. \{\nu_{g1}^*, \nu_{g2}^*, \nu_{g3}^*, \nu_{g4}^*\}, \{c_{g1}^*, c_{g2}^*, c_{g3}^*, c_{g4}^*\} \right\}, \\ \Theta_h &= \left\{ \{\bar{h}^{*,1}, \delta_{h,g1}^{*,1}, \delta_{h,d}^{*,1}\}, \{\bar{h}^{*,2}, \delta_{h,g1}^{*,2}, \delta_{h,g2}^{*,2}, \delta_{h,d}^{*,2}\}, \{\bar{h}^{*,3}, \delta_{h,g1}^{*,3}, \delta_{h,g2}^{*,3}, \delta_{h,d}^{*,3}\}, \dots \right. \\ &\quad \left. \{\bar{h}^{*,4}, \delta_{h,g1}^{*,4}, \delta_{h,g3}^{*,4}, \delta_{h,d}^{*,4}\}, \{\bar{h}^{*,5}, \delta_{h,g1}^{*,5}, \delta_{h,g3}^{*,5}, \delta_{h,d}^{*,5}\}, \{\bar{h}^{*,6}, \delta_{h,g1}^{*,6}, \delta_{h,g3}^{*,6}, \delta_{h,d}^{*,6}\} \right\}, \\ \Theta_s &= \left\{ \{\bar{s}^*, \delta_{s,g4}^*, \theta^*\}, \{\bar{s}, \delta_{s,g4}, \bar{v}, \delta_{v,g4}, \theta\} \right\}, \\ \Theta_d &= \left\{ \{\mu_d, \delta_{d,g1}\}, \{\Phi_d, \rho_d^*\} \right\}\end{aligned}$$

The number of parameters are as follows:

- The model with contagion has a total of 55 parameters $\#\Theta_g = 20, \#\Theta_h = 23, \#\Theta_s = 8, \#\Theta_d = 4$.
- The model without contagion has a total of 47 parameters $\#\Theta_g = 20, \#\Theta_h = 17, \#\Theta_s = 8, \#\Theta_d = 2$. Here, we are removing $\delta_{h,d}^{*,k}$ for $k \in \{1, \dots, 6\}$ and $\{\Phi_d, \rho_d^*\}$.

It is important to mention that parameters associated with the interest rate factors

$$\Theta_u = \{\mu_{u1}^*, \mu_{u2}^*, \phi_{u11}^*, \phi_{u21}^*, \phi_{u22}^*, \bar{r}, \delta_{u1}, \delta_{u2}\}$$

are not estimated and provided from the first stage interest rate estimation. We use a Bayesian approach to make joint inference about parameters $\Theta = \{\Theta_g, \Theta_h, \Theta_s, \Theta_d\}$ and the latent state vector x_t in equation (22). Bayesian inference requires the specification of a prior distribution $p(\Theta)$ and the evaluation of the likelihood function $p(Y|\Theta)$. Most of our priors are noninformative. We use MCMC methods to generate a sequence of draws $\{\Theta^{(j)}\}_{j=1}^{n_{sim}}$ from the posterior distribution $p(\Theta|Y) = \frac{p(Y|\Theta)p(\Theta)}{p(Y)}$. The numerical evaluation of the prior density and the likelihood function $p(Y|\Theta)$ is done with the particle filter.

Given (A), (B), (C), we use a particle-filter approximation of the likelihood function (28) and embed this approximation into a fairly standard random walk Metropolis algorithm. See [Herbst and Schorfheide \(2016\)](#) for a review of particle filter.

In the subsequent exposition we omit the dependence of all densities on the parameter vector Θ . The particle filter approximates the sequence of distributions $\{p(x_t|Y_{1:t})\}_{t=1}^T$ by

a set of pairs $\{x_t^{(i)}, \pi_t^{(i)}\}_{i=1}^N$, where $x_t^{(i)}$ is the i th particle vector, $\pi_t^{(i)}$ is its weight, and N is the number of particles. As a by-product, the filter produces a sequence of likelihood approximations $\hat{p}(y_t|Y_{1:t-1})$, $t = 1, \dots, T$.

- Initialization: We generate the particle values $x_0^{(i)}$ from the unconditional distribution. We set $\pi_0^{(i)} = 1/N$ for each i .
- Propagation of particles: We simulate (22) forward to generate $x_t^{(i)}$ conditional on $x_{t-1}^{(i)}$. We use $q(x_t|x_{t-1}^{(i)}, y_t)$ to represent the distribution from which we draw $x_t^{(i)}$.
- Correction of particle weights: Define the unnormalized particle weights for period t as

$$\tilde{\pi}_t^{(i)} = \pi_{t-1}^{(i)} \times \frac{p(y_t|x_t^{(i)})p(x_t^{(i)}|x_{t-1}^{(i)})}{q(x_t^{(i)}|x_{t-1}^{(i)}, y_t)}. \quad (31)$$

The term $\pi_{t-1}^{(i)}$ is the initial particle weight and the ratio $\frac{p(y_t|x_t^{(i)})p(x_t^{(i)}|x_{t-1}^{(i)})}{q(x_t^{(i)}|x_{t-1}^{(i)}, y_t)}$ is the importance weight of the particle. The last equality follows from the fact that we chose $q(x_t^{(i)}|x_{t-1}^{(i)}, y_t) = p(x_t^{(i)}|x_{t-1}^{(i)})$.

The log likelihood function approximation is given by

$$\log \hat{p}(y_t|Y_{1:t-1}) = \log \hat{p}(y_{t-1}|Y_{1:t-2}) + \log \left(\sum_{i=1}^N \tilde{\pi}_t^{(i)} \right).$$

- Resampling: Define the normalized weights

$$\pi_t^{(i)} = \frac{\tilde{\pi}_t^{(i)}}{\sum_{j=1}^N \tilde{\pi}_t^{(j)}}$$

and generate N draws from the distribution $\{x_t^{(i)}, \pi_t^{(i)}\}_{i=1}^N$ using multinomial resampling. In slight abuse of notation, we denote the resampled particles and their weights also by $x_t^{(i)}$ and $\pi_t^{(i)}$, where $\pi_t^{(i)} = 1/N$.

Table 1
Affine Term Structure Models of Sovereign Credit Spreads

Study	Model Focus			Default Intensity Components			FX Components			Factors			
	TS	FX	Quanto	Homoscedastic	Heteroscedastic	Contagion	Homoscedastic	Heteroscedastic	Extr. Evts	Country	Common	Region	Joint Est.
Monfort and Renne (2014)				✓						✓			
Duffie et al. (2003)				✓	✓					✓			
Hoerdahl and Tristani (2012)	✓			✓	✓					✓	✓		✓
Pan and Singleton (2008)	✓			✓						✓			
Longstaff et al. (2011)	✓*			✓						✓			
Doshi et al. (2017)	✓			✓						✓	✓		✓
Zhang (2008)	✓			✓	✓					✓			
Ang and Longstaff (2013)	✓*			✓	✓					✓	✓		
Aït-Sahalia et al. (2014)	✓**					✓				✓			+
Benzoni et al. (2015)				✓		✓				✓	✓		✓
Carr and Wu (2007)	✓*	✓		✓	✓		✓	✓	✓	✓			
Ehlers and Schönbucher (2004)		✓	✓	✓			✓		✓	✓			
Brigo et al. (2016)	✓**	✓	✓	✓			✓		✓	✓			
Monfort et al. (2017)	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓		
The present study	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓

Notes. This table summarizes the main affine term structure models proposed for the pricing of sovereign credit spreads using intensity-based frameworks. We describe the focus of the paper, which can encompass the term structure (TS), foreign exchange rates (FX), and CDS quantos (Quanto). We also indicate the main model components of the default intensity, and, if applicable, of the depreciation rate dynamics. We refer to the presence of homoscedastic or heteroscedastic shocks, extreme events, and contagion. We further describe the type of risk factors, which can be country-specific, regional, or common. Finally, we indicate whether the estimation is done jointly for all countries, or on a country-by-country basis. + indicates that the estimation is done pairwise for two countries. The * refers to the fact that the estimation is performed on the short end of the term structure, up to the 5-year maturity. ** denotes that the estimation considers only two maturity segments, 5 and 10 years.

Table 2
Descriptive Statistics of CDS Quanto Spreads - Weekly

Country	Maturity Obs	1y		3y		5y		7y		10y		15y		30y		10y-1y	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
AT	329	8	11	14	14	21	17	25	17	29	17	33	17	34	16	21	10
BE	329	11	16	20	21	28	22	33	22	37	21	39	21	40	21	26	14
CY	329	28	81	28	52	29	41	30	40	33	40	34	41	40	56	5	73
EE	329	3	6	5	5	6	5	6	5	7	5	7	6	7	7	3	5
FI	329	3	3	5	3	8	3	11	4	13	4	14	5	14	6	10	5
FR	329	8	11	17	18	27	23	32	24	37	23	38	23	40	24	29	16
DE	329	3	5	8	8	16	13	20	14	24	14	26	14	26	15	21	12
GR	255	127	454	91	264	90	227	88	223	80	228	74	168	59	130	-29	318
IE	329	21	24	29	26	34	26	36	25	38	24	40	24	40	24	18	13
IT	329	21	20	31	23	36	24	39	24	42	24	43	24	45	24	21	9
LV	329	6	9	9	9	12	9	13	8	15	8	18	8	17	10	10	7
LT	329	6	9	10	9	13	8	16	7	17	8	19	8	20	10	11	6
NL	329	5	6	10	8	17	11	21	12	25	13	27	15	28	15	20	10
PT	329	34	36	37	27	40	24	42	22	44	21	42	21	42	25	10	24
SK	329	6	9	7	7	8	7	9	8	10	8	13	9	13	10	4	9
SI	329	10	13	14	13	17	13	19	14	21	15	24	16	25	17	10	12
ES	329	27	28	38	33	44	33	47	33	50	32	53	31	53	31	22	11
Total	5,519	13	72	18	104	21	63	25	56	28	55	30	55	33	40	32	33

Notes. This table reports summary statistics (mean, sd) for the sovereign CDS quanto spreads (difference between the USD and EUR denominated CDS spreads) for 17 Eurozone that have a minimum of 365 days of non-zero information on USD-EUR CDS quanto spreads. We report values for maturities of 1y, 3y, 5y, 7y, 10y, 15y, and 30y, as well as the slope, defined as the difference between the 10y and 1y quanto spreads. The sample period is August 20, 2010 until December 30, 2016. The data frequency is weekly, based on Wednesday quotes. Source: Markit.

Table 3
Descriptive Statistics of Sovereign CDS Liquidity - Weekly

	N	Mean	EUR			Mean	USD			Gross	USD Notionals		
			Min	p50	Max		Min	p50	Max		Gross%	Net	Net%
Austria	329	4.50	3	4	8	6.21	3	6	9	54.37	15.51	5.04	23.35
Belgium	329	4.73	3	5	8	5.55	3	5	10	54.96	15.55	4.50	20.72
Cyprus	329	2.75	2	2	6	3.69	2	3	9	1.88	0.48	0.25	1.24
Estonia	329	3.45	2	3	8	4.02	3	4	8	2.51	0.74	0.27	1.25
Finland	329	3.15	2	3	9	5.79	2	6	9	17.48	4.98	2.17	10.28
France	329	4.77	2	5	9	5.72	2	6	9	136.17	37.96	15.72	73.51
Germany	329	3.89	2	3	8	5.19	2	5	9	121.24	33.97	14.99	70.54
Greece	255	3.12	2	3	7	3.85	2	4	7	48.75	16.97	3.43	13.89
Ireland	329	5.14	3	5	9	6.60	3	7	10	44.36	12.72	2.80	12.82
Italy	329	6.38	2	7	9	6.61	2	7	10	356.35	100.00	21.43	100.00
Latvia	329	4.02	2	4	10	5.32	3	5	10	8.96	2.58	0.50	2.34
Lithuania	329	3.68	2	4	7	4.47	2	5	9	6.53	1.86	0.48	2.24
Netherlands	326	3.60	2	3	8	5.27	2	5	9	28.38	7.94	3.00	14.28
Portugal	329	5.78	3	6	9	7.24	2	7	10	69.48	19.91	4.41	20.14
Slovakia	329	3.81	2	4	10	5.42	3	5	10	10.73	3.06	0.75	3.53
Slovenia	329	3.21	2	3	7	4.27	2	4	9	6.96	1.93	0.68	3.19
Spain	329	5.66	2	5	10	6.10	2	6	10	171.38	48.47	12.52	57.98
All	5,516	4.23	2	4	10	5.39	2	5	10	69.05	19.56	5.64	26.24

Notes. This table reports summary statistics (mean, N, min, max, median) for the depth measure of 5-year sovereign CDS spreads denominated in EUR and USD for 17 Eurozone countries that have a minimum of 365 days of non-zero information on USD-EUR quanto CDS spreads. A simple *t*-test for differences in means suggests that differences in means are statistically significant. Depth is defined as the number of dealers used in the computation of the daily mid market CDS quote. The sample period is August 20, 2010 until December 30, 2016. All statistics are based on weekly (Wednesday) data. We also report the gross and net notional amounts of CDS outstanding in billion USD, as well as the ratio of net and gross notional amounts of CDS outstanding to the same quantities of Italy, which represents the most liquid sovereign CDS market. This data is based on weekly information from August 20, 2010 until June 24, 2015. Sources: Markit and Depository Trust and Clearing Corporation (DTCC).

Table 4
Parameter estimates: Model of the OIS term structure

	5%	50%	95%		5%	50%	95%
μ_{u1}^*	-1.0098	-0.8119	-0.6651	$\bar{r}$	0.0007	0.0026	0.0050
μ_{u2}^*	0.0540	0.1027	0.1464	δ_{u1}	0.0000	0.0001	0.0002
-	-	-	-	δ_{u2}	0.0016	0.0019	0.0021
ϕ_{u11}^*	0.9985	0.9987	0.9990	ϕ_{u11}	0.9761	0.9767	0.9776
ϕ_{u12}^*	-	-	-	ϕ_{u12}	-0.0815	-0.0808	-0.0800
ϕ_{u21}^*	-0.0024	-0.0022	-0.0020	ϕ_{u21}	-0.0002	0.0013	0.0026
ϕ_{u22}^*	0.9984	0.9986	0.9987	ϕ_{u22}	0.9560	0.9568	0.9595

Notes. In this table, we report the parameter estimates for the OIS term structure. The model is estimated using Bayesian MCMC. We report the posterior medians, as well as the 5th and 95th percentiles of the posterior distribution. The sample period is August 20, 2010 to December 30, 2016. The data frequency is weekly, based on Wednesday rates.

Table 5
Parameter estimates: Model without contagion

	5%	50%	95%		5%	50%	95%
(A) factor dynamics							
ϕ_{11}^*	0.9986	0.9988	0.9992	ϕ_{11}	0.9258	0.9668	0.9860
ϕ_{21}^*	0.0045	0.0047	0.0050	ϕ_{21}	0.0006	0.0211	0.0915
ϕ_{22}^*	0.9981	0.9985	0.9990	ϕ_{22}	0.8867	0.9640	0.9894
ϕ_{31}^*	-0.0036	-0.0033	-0.0030	ϕ_{31}	-0.0206	0.0160	0.0552
ϕ_{33}^*	0.9976	0.9982	0.9986	ϕ_{33}	0.9001	0.9554	0.9772
ϕ_{44}^*	0.9997	0.9999	0.9999	ϕ_{44}	0.8288	0.8575	0.9664
c_1^*	0.0063	0.0068	0.0072	ν_1	1.7465	1.8693	2.0231
c_2^*	0.0096	0.0100	0.0117	ν_2	0.1915	0.2934	0.3475
c_3^*	0.0073	0.0104	0.0110	ν_3	1.6170	1.7630	1.8348
c_4^*	0.0041	0.0047	0.0067	ν_4	0.8274	0.8587	0.9027
(B) hazard rates							
$10000 \times \bar{h}^{*1}$	0.3011	0.4289	0.4833	$10000 \times \bar{h}^{*2}$	0.1451	0.1901	0.2637
δ_{w1}^{*1}	0.0011	0.0014	0.0018	δ_{w1}^{*2}	0.0018	0.0022	0.0027
-	-	-	-	δ_{w2}^{*2}	0.0034	0.0041	0.0045
$10000 \times \bar{h}^{*3}$	0.5941	0.6717	0.7029	$10000 \times \bar{h}^{*4}$	0.0024	0.0280	0.0649
δ_{w1}^{*3}	0.0014	0.0022	0.0027	δ_{w1}^{*4}	0.0026	0.0032	0.0037
δ_{w2}^{*3}	0.0017	0.0025	0.0028	δ_{w3}^{*4}	0.0020	0.0027	0.0033
$10000 \times \bar{h}^{*5}$	0.1847	0.2590	0.3535	$10000 \times \bar{h}^{*6}$	0.1037	0.1523	0.1835
δ_{w1}^{*5}	0.0028	0.0037	0.0043	δ_{w1}^{*6}	0.0016	0.0021	0.0027
δ_{w3}^{*5}	0.0017	0.0025	0.0033	δ_{w3}^{*6}	0.0034	0.0044	0.0050
(C) exchange rates							
$\bar{s}^*$	0.0052	0.0061	0.0075	$\bar{s}$	0.0001	0.0012	0.0046
δ_{s6}^*	-0.0061	-0.0056	-0.0049	δ_{s6}	-0.0152	-0.0122	-0.0006
-	-	-	-	$\bar{v}$	0.0004	0.0004	0.0005
-	-	-	-	δ_v	0.0002	0.0003	0.0004
θ^*	0.0421	0.0590	0.0771	θ	0.0011	0.0085	0.0211
(D) default intensity							
$10000 \times \bar{h}$	0.3120	0.5204	0.8729				
δ_{w1}	0.0051	0.0089	0.0114				

Notes. In this table, we report the parameter estimates for the CDS quanto model without contagion. The model is estimated using Bayesian MCMC. We report the posterior medians, as well as the 5th and 95th percentiles of the posterior distribution. In Panel A, we report estimates for the credit and volatility factors. In Panel B we report estimates for the hazard rates. The superscripts in default intensity parameters refer to countries in the following order: Germany, Belgium, France, Ireland, Italy, Portugal. In Panel C, we report estimates for the exchange rate dynamics. In Panel D, we report estimates for the aggregate physical default intensity.

Table 6
Parameter estimates: Model with contagion

	5%	50%	95%		5%	50%	95%
(A) factor dynamics							
ϕ_{11}^*	0.9990	0.9995	0.9997	ϕ_{11}	0.9572	0.9760	0.9909
ϕ_{21}^*	0.0048	0.0050	0.0055	ϕ_{21}	0.0174	0.0387	0.0980
ϕ_{22}^*	0.9975	0.9978	0.9983	ϕ_{22}	0.8885	0.9400	0.9734
ϕ_{31}^*	-0.0036	-0.0033	-0.0031	ϕ_{31}	-0.0079	0.0367	0.0668
ϕ_{33}^*	0.9977	0.9979	0.9980	ϕ_{33}	0.8645	0.9198	0.9821
ϕ_{44}^*	0.9990	0.9993	0.9999	ϕ_{44}	0.8658	0.9149	0.9518
c_1^*	0.0060	0.0066	0.0071	ν_1	1.6750	1.7353	1.8213
c_2^*	0.0101	0.0115	0.0127	ν_2	0.0624	0.1072	0.1751
c_3^*	0.0061	0.0071	0.0077	ν_3	1.6774	1.7137	1.7423
c_4^*	0.0068	0.0080	0.0086	ν_4	0.8123	0.8558	0.9251
(B) hazard rates							
$10000 \times \bar{h}^{*1}$	0.2575	0.3161	0.4170	$10000 \times \bar{h}^{*2}$	0.0230	0.2003	0.2809
δ_{w1}^{*1}	0.0011	0.0014	0.0016	δ_{w1}^{*2}	0.0010	0.0016	0.0022
-	-	-	-	δ_{w2}^{*2}	0.0030	0.0036	0.0041
δ_d^{*1}	0.0007	0.0037	0.0122	δ_d^{*2}	0.0173	0.0361	0.0729
$10000 \times \bar{h}^{*3}$	0.5029	0.6049	0.6239	$10000 \times \bar{h}^{*4}$	0.0099	0.0259	0.0785
δ_{w1}^{*3}	0.0011	0.0012	0.0016	δ_{w1}^{*4}	0.0026	0.0033	0.0039
δ_{w2}^{*3}	0.0022	0.0025	0.0031	δ_{w3}^{*4}	0.0023	0.0031	0.0037
δ_d^{*3}	0.0145	0.0270	0.0459	δ_d^{*4}	0.0016	0.0178	0.0370
$10000 \times \bar{h}^{*5}$	0.1574	0.1872	0.2543	$10000 \times \bar{h}^{*6}$	0.1056	0.1662	0.2346
δ_{w1}^{*5}	0.0027	0.0037	0.0045	δ_{w1}^{*6}	0.0011	0.0018	0.0029
δ_{w3}^{*5}	0.0020	0.0030	0.0034	δ_{w3}^{*6}	0.0044	0.0050	0.0061
δ_d^{*5}	0.0004	0.0107	0.0284	δ_d^{*6}	0.0037	0.0149	0.0257
(C) exchange rates							
$\bar{s}^*$	0.0067	0.0075	0.0089	$\bar{s}$	0.0001	0.0012	0.0066
δ_{s6}^*	-0.0105	-0.0079	-0.0060	δ_{s6}	-0.0182	-0.0120	-0.0004
-	-	-	-	$\bar{v}$	0.0000	0.0001	0.0004
-	-	-	-	δ_v	0.0004	0.0012	0.0016
θ^*	0.0526	0.0612	0.0739	θ	0.0009	0.0082	0.0155
(D) default intensity							
$\bar{h}$	0.0000	0.0001	0.0001	ρ^*	1.0619	1.4039	2.0572
δ_{w1}	0.0082	0.0094	0.0122	δ_d	0.0000	0.0002	0.0004

Notes. In this table, we report the parameter estimates for the CDS quanto model with contagion. The model is estimated using Bayesian MCMC. We report the posterior medians, as well as the 5th and 95th percentiles of the posterior distribution. In Panel A, we report estimates for the credit and volatility factors. In Panel B we report estimates for the hazard rates. The superscripts in default intensity parameters refer to countries in the following order: Germany, Belgium, France, Ireland, Italy, Portugal. In Panel C, we report estimates for the exchange rate dynamics. In Panel D, we report estimates for the aggregate physical default intensity.

Table 7
Model comparison

	With contagion			Without contagion		
	5%	50%	95%	5%	50%	95%
$\ln p(Y \Theta)$	78339	78581	78794	78313	78567	78778
BIC	-78174	-78416	-78629	-78171	-78425	-78636

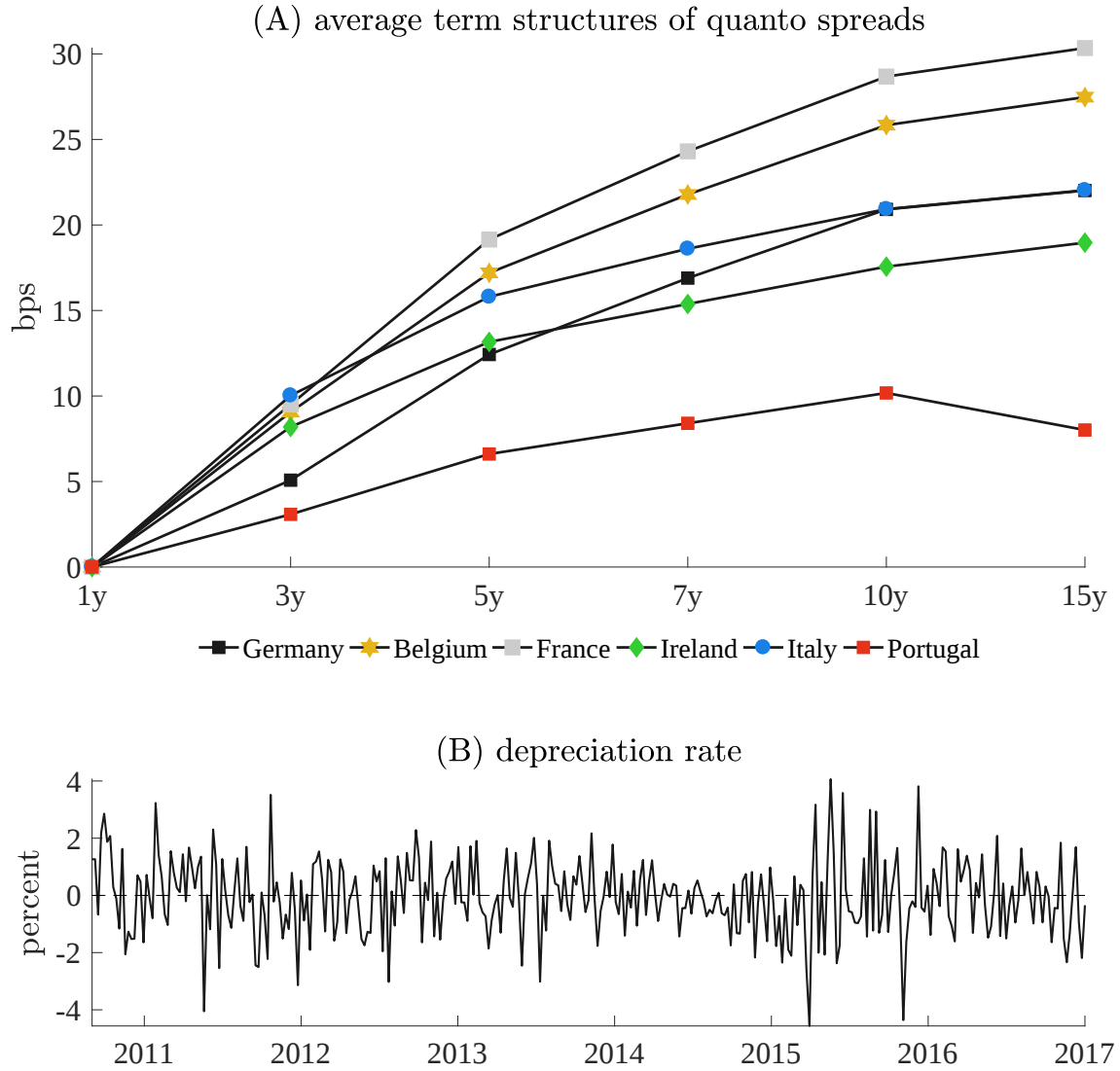
Notes. In this table, we report the distributions of the likelihoods of both models, and the associated Bayesian Information Criteria (negative of the likelihood plus penalty for the number of parameters). The model is estimated using Bayesian MCMC. We report the posterior medians, as well as the 5th and 95th percentiles of the posterior distribution. The model with the lowest Bayesian Information Criterion (BIC) is preferred.

Table 8
Model fit

	Bond yield			Quanto spread				Avg across countries
		Germany	Belgium	France	Ireland	Italy	Portugal	
1y	9.0	2.2	3.8	3.6	13.6	8.0	24.5	9.3
3y	26.2	3.4	6.1	5.2	8.1	4.7	8.9	6.1
5y	25.5	4.0	6.9	5.8	7.8	4.4	5.7	5.8
7y	19.9	3.8	4.3	5.7	7.1	4.6	5.4	5.1
10y	17.7	4.7	4.5	6.7	7.0	5.6	7.1	5.9
15y	-	5.5	5.5	7.6	8.4	6.5	13.6	7.8
Avg across maturity	14.0	3.9	5.2	5.8	8.7	5.6	10.8	-

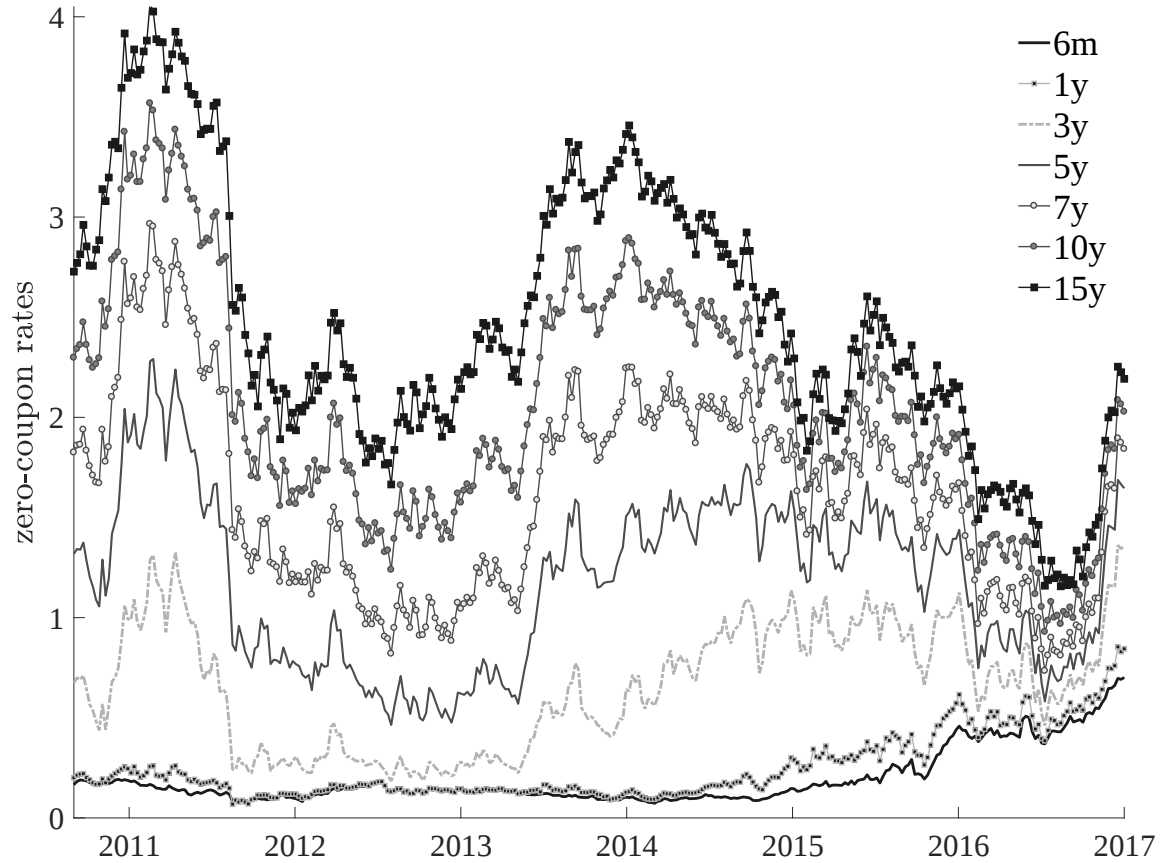
Notes. In this table, we report results for the model fit in terms of root mean squared errors (RMSE). For the term structure model, we do not allow for measurement errors for 6-month- and 15-year maturity bonds in the estimation. We report the RMSE in basis points. The sample period is August 20, 2010 to December 30, 2016. The data frequency is weekly, based on Wednesday rates.

Figure 1
Quanto term spreads and exchange rate



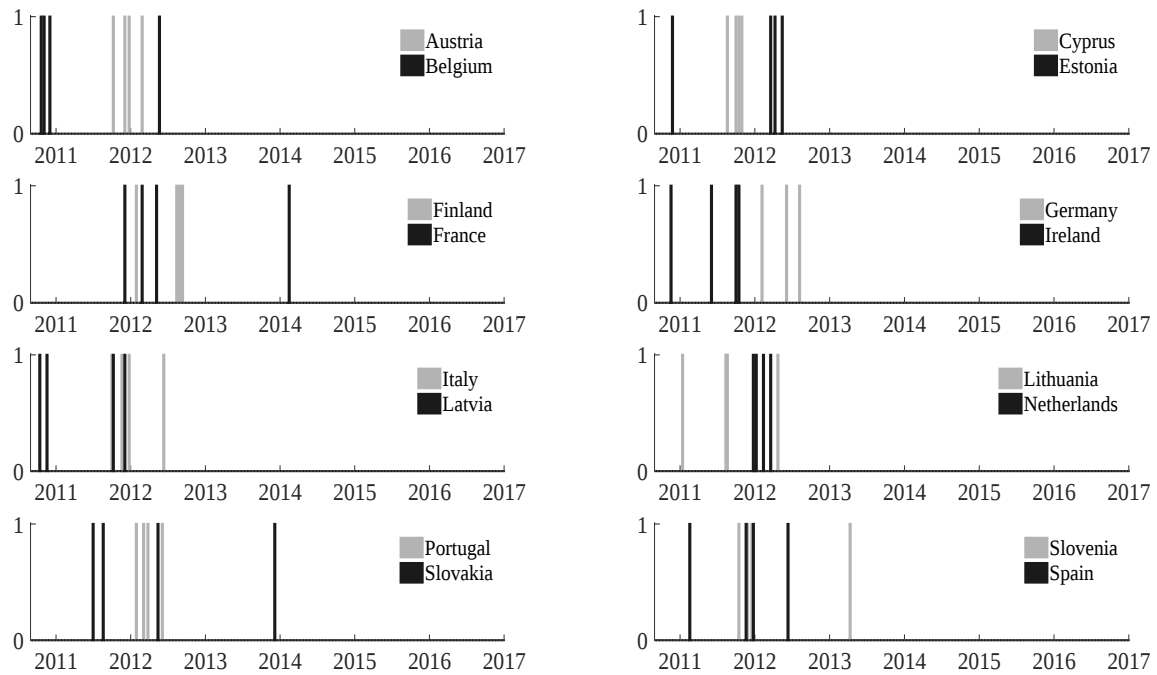
Notes. Panel A displays the the term structure of CDS quanto spreads, defined as the difference between the USD and EUR denominated CDS spreads, for 6 Eurozone countries: Germany , Belgium, France, Ireland, Italy, Portugal. We compute average term spreads of maturities 3 years, 5 years, 7 years, 10 years, 15 years, and 30 years relative to the 1 year quanto spread. All spreads are expressed in basis points. Panel B shows weekly Wednesday-to-Wednesday changes in the USD/EUR exchange rate, expressed in percentage point changes. The sample period is August 20, 2010 to December 30, 2016.

Figure 2
Time series of the OIS interest rates



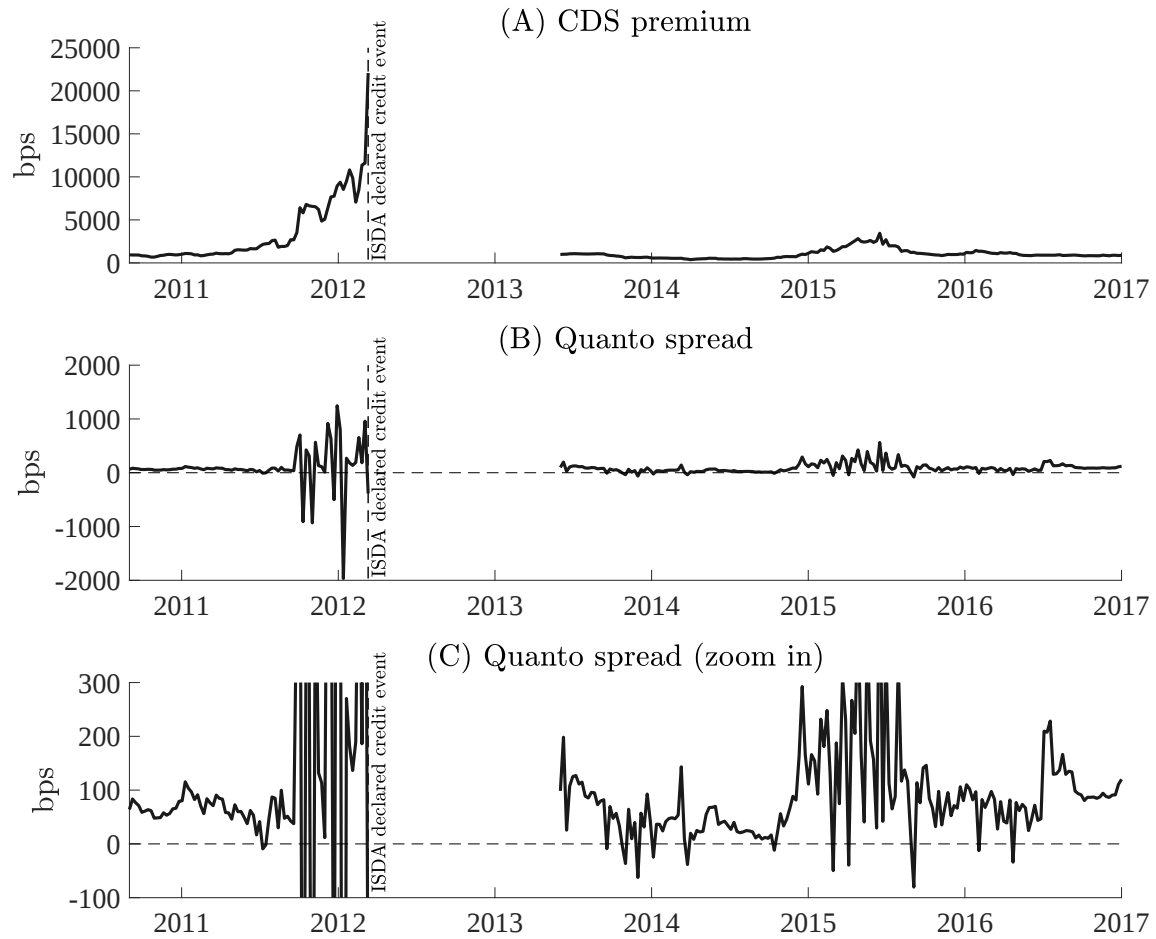
Notes. The Figure displays zero-coupon rates bootstrapped from the term structure of interest rates. We use weekly observations for Overnight index swap (OIS) rates with maturities of 1, 3, 6, 9, 12, 36, and 60 months to maturity. For interest rate swap rates, we use weekly observations with maturities of 7, 10, 15, and 30 years. Zero-coupon rates for interest rates of five years and lower are bootstrapped from OIS spreads. For maturities above five years, we bootstrap zero-coupon rates from IRS rates, but discount the implied zero-coupon rates by an amount equivalent to the difference in zero coupon rates obtained from the IRS and OIS curve. The sample period is August 20, 2010 through December 30, 2016.

Figure 3
Time series of credit events



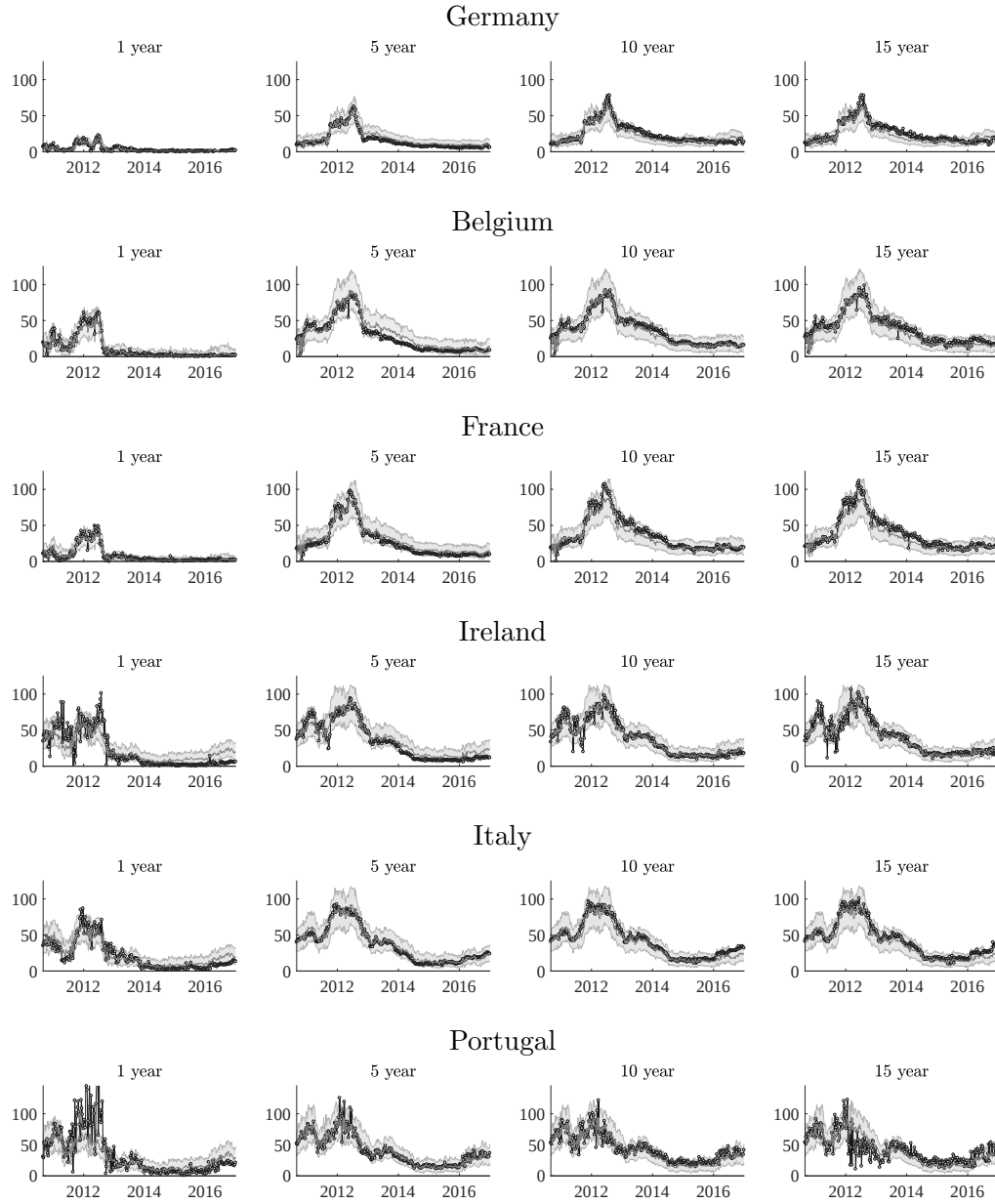
Notes. These figures depict the time series of credit events for 16 Eurozone countries that have a minimum of 365 days of non-zero information on USD-EUR quanto CDS spreads. Greece is omitted from this figure. In the absence of actual credit events, we define them as occurrences when a 5-year quanto spread is above the 99th percentile of the country-specific distribution of quanto spread changes.

Figure 4
5-year CDS premiums and Quanto spreads of Greece



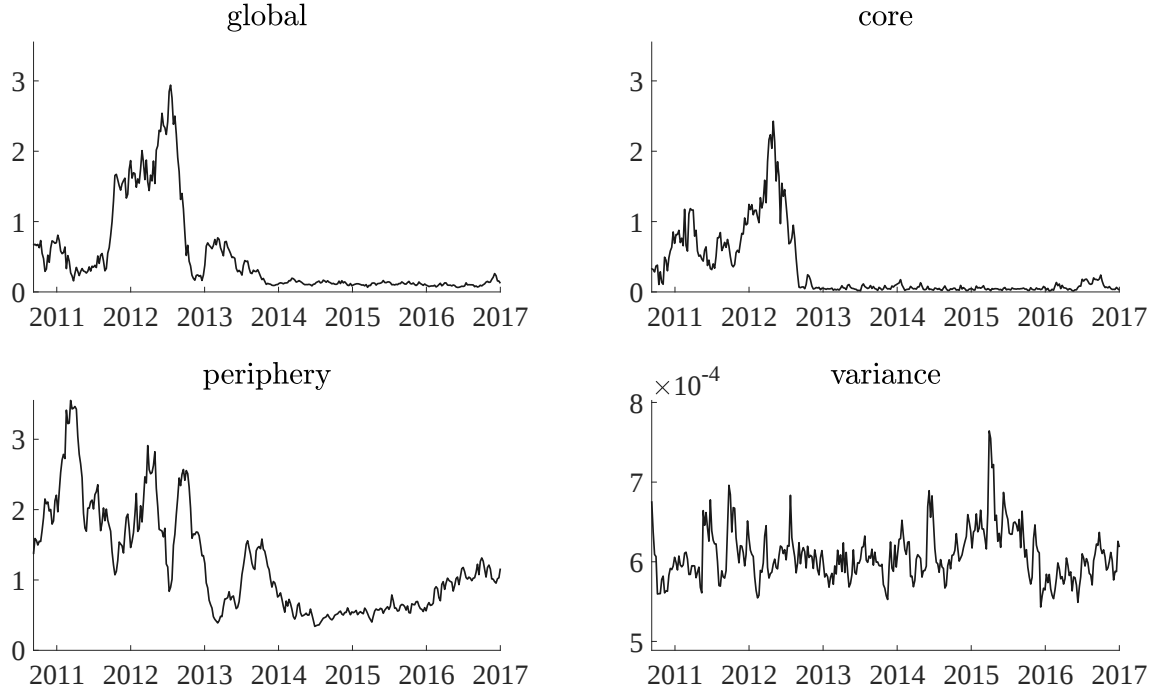
Notes. Panel A depicts the 5-year USD CDS premium (in bps). Panel B depicts the 5-year USD-EUR CDS quanto spread (in bps) for Greece. The sample period is August 20, 2010 to December 30, 2016. No data is available between March 8, 2012 and June 10, 2013.

Figure 5
Quanto spread



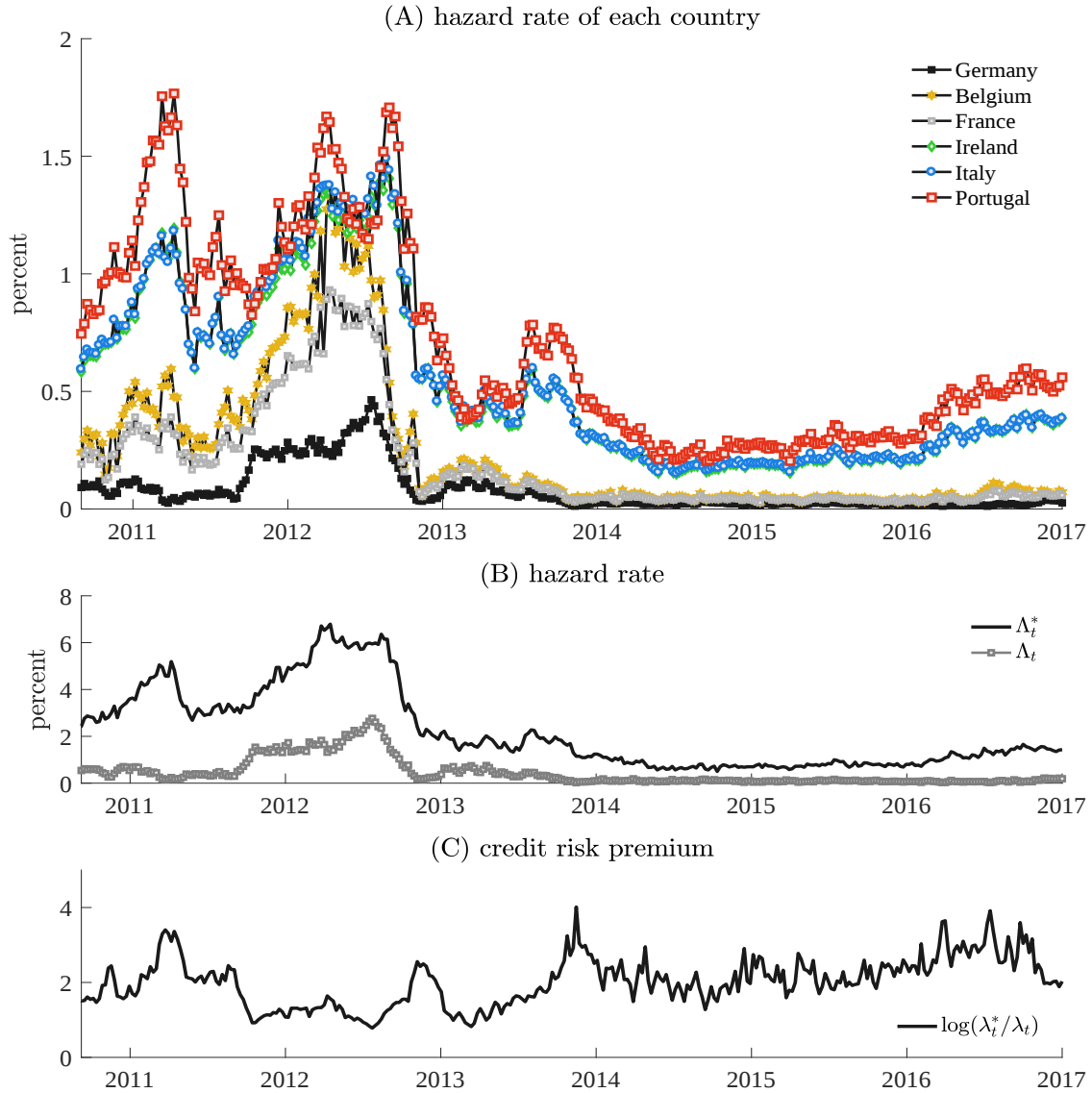
Notes. In these figures, we plot the observed and model-implied USD/EUR quanto spreads for Germany, Belgium, France, Ireland, Italy, and Portugal. We report values for maturities of 1y, 5y, 10y, and 15y. Gray lines represent posterior medians of quanto spreads and gray-shaded areas correspond to 90% credible intervals. The actual quanto spread are plotted with black-circled lines.

Figure 6
State



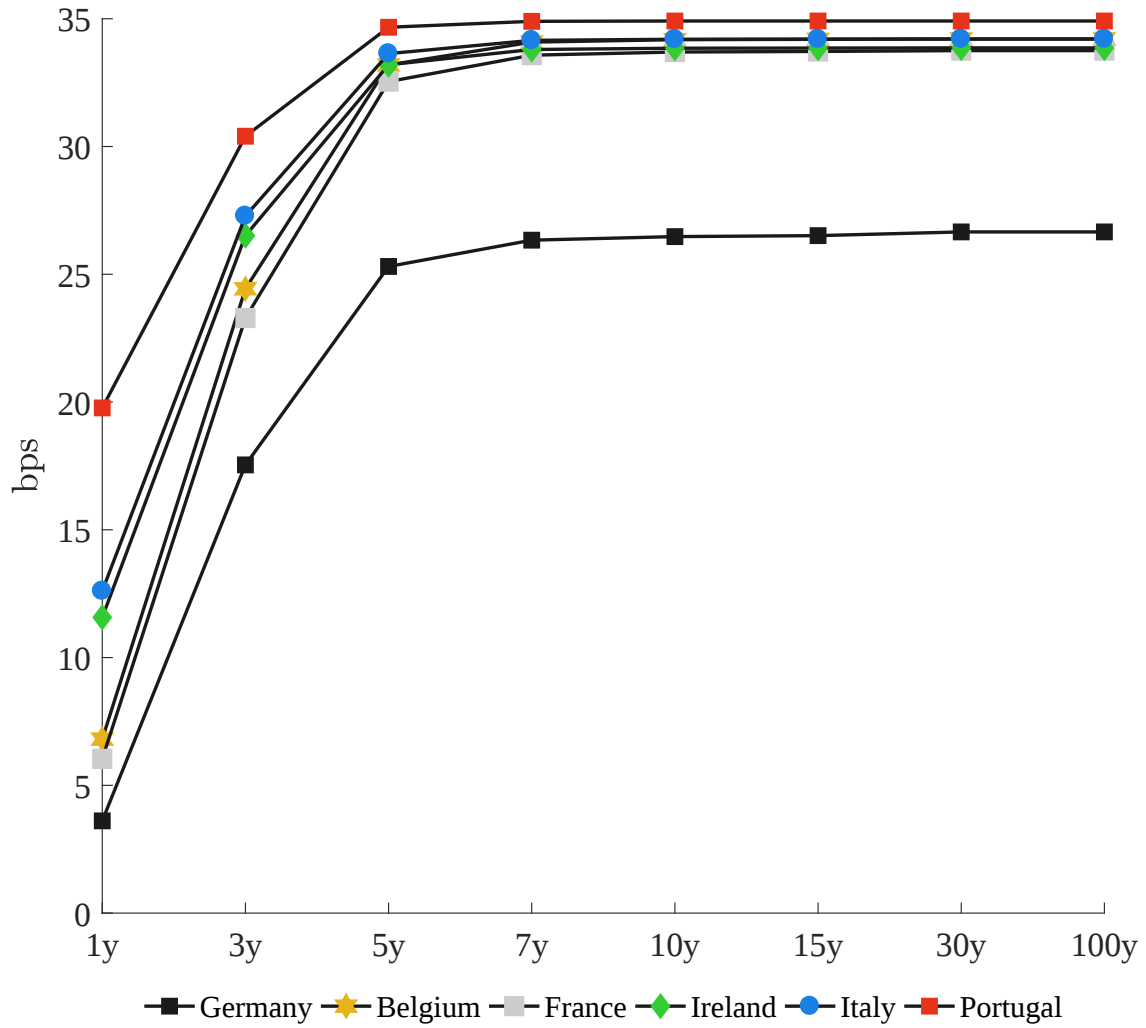
Notes. These figures depict the filtered latent credit and volatility factors implied by the model. w_{1t} is the global credit factor, w_{2t} is a regional credit factor corresponding to the core countries in the Eurozone (Belgium and France), and w_{3t} is a regional credit factor corresponding to the peripheral countries (Ireland, Italy, Portugal). The factor v_t controls variance of the (log) depreciation rate. The sample period is August 20, 2010 until December 30, 2016. The data frequency is weekly.

Figure 7
Hazard rates



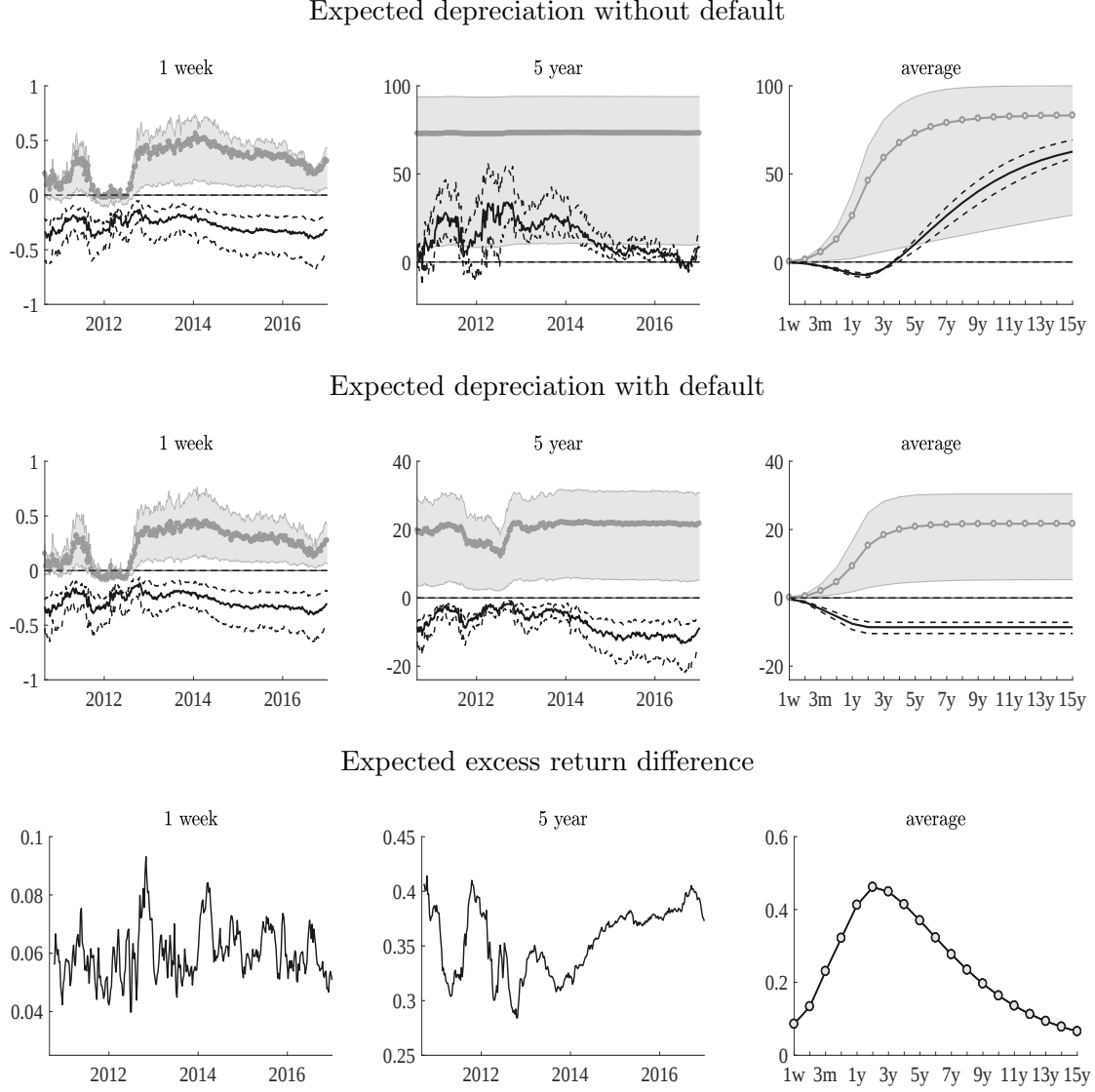
Notes. Panel A depicts the estimated risk-neutral hazard rates for Germany, Belgium, France, Ireland, Italy, and Portugal. In Panel B, we plot the aggregate and actual hazard rates. In Panel C, we plot the default risk premium, defined as $\log(\lambda_t^*/\lambda_t)$. The sample period is August 20, 2010 until December 30, 2016. The data frequency is weekly, based on Wednesday quotes.

Figure 8
Average term structure of quanto spreads



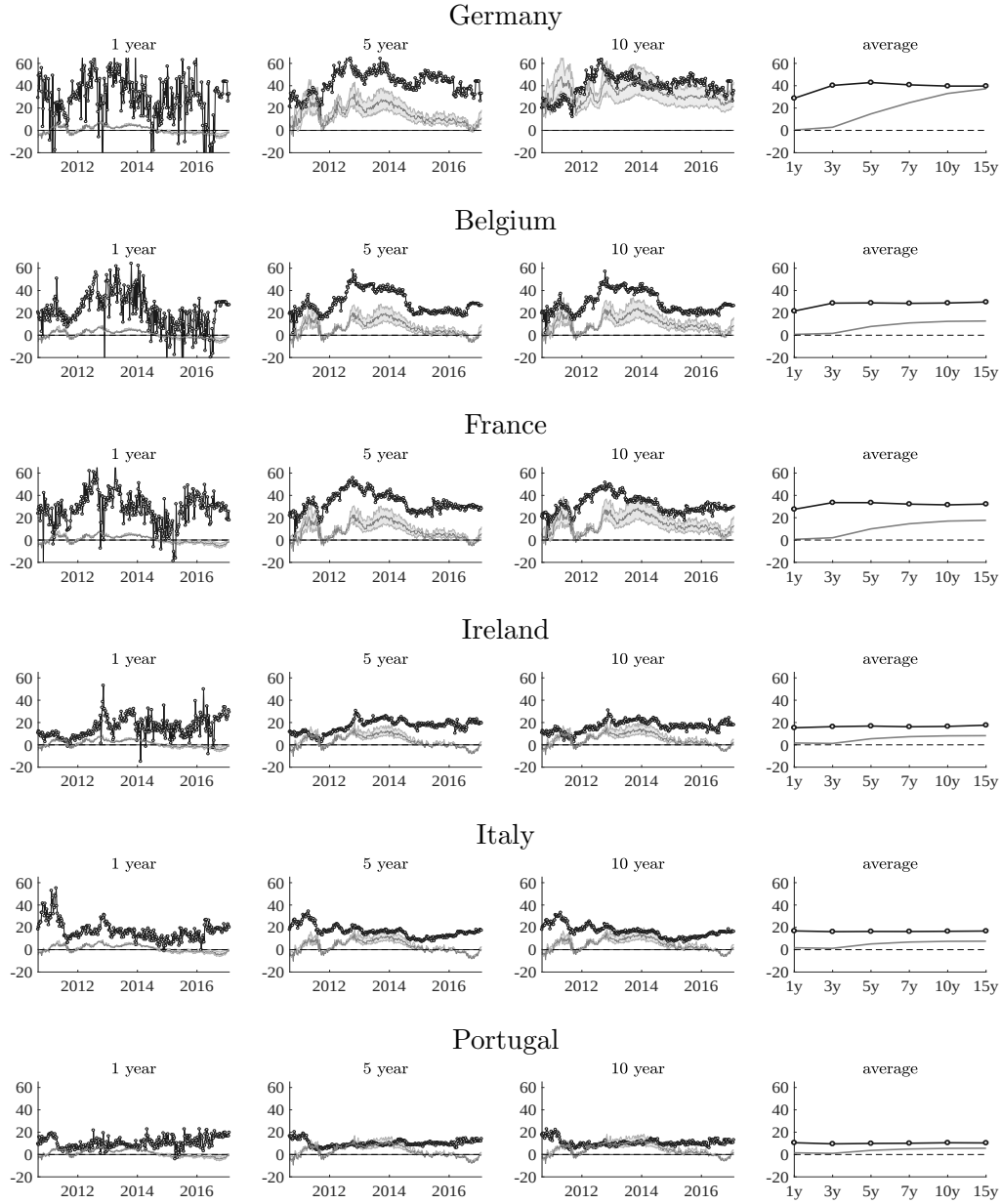
Notes. This figure depicts the model-implied average term structure of USD/EUR CDS quanto spreads for Germany, Belgium, France, Ireland, Italy, and Portugal. We report values for maturities of 1 year to 100 years. The model is estimated using data from August 20, 2010 until December 30, 2016. The data frequency is weekly, based on Wednesday quotes.

Figure 9
Objective and risk-neutral expectations of the depreciation rate



Notes. These figures depict the objective (light gray) and risk-neutral (black) expectations of the depreciation rate. The first row shows the expectations ignoring default, $E_t[1 - S_T/S_t]$ and $E_t^*[1 - S_T/S_t]$. The second row accounts for default, $E_t[1 - S_{\tau \wedge T}/S_t]$ and $E_t^*[1 - S_{\tau \wedge T}/S_t]$. The credit event time τ is triggered by a credit event in any of the countries. The left and middle panels show time-series for $T = 1$ week and 5 years, respectively. The right panels display time-series averages of these quantities for a variety of T 's. The last row computes the difference between excess log expected returns corresponding to the two scenarios: $(T - t)^{-1} \log[E_t S_{\tau \wedge T}/E_t^* S_{\tau \wedge T} \cdot E_t^* S_T/E_t S_T]$ – a measure of premium for the risk of currency devaluation at default.

Figure 10
Relative quanto spread and expected depreciation rate



Notes. In this figure, we plot the observed relative quanto spreads (black-circled lines) and the model-implied expected depreciation rate $E_t[1 - S_{\tau \wedge T}/S_t]$ (gray, gray-shaded areas correspond to 90% credible intervals) for $T = 1, 3, 5, 7, 10$, and 15 years, together with their sample averages in the last column. The results are for Germany, Belgium, France, Ireland, Italy, and Portugal. The model is estimated using data from August 20, 2010 until December 30, 2016. The data frequency is weekly, based on Wednesday quotes.