Valuing Subscription-Based Businesses Using Publicly Disclosed Customer Data

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Abstract

Valuing Subscription-Based Businesses Using Publicly Disclosed Customer Data

The growth of subscription-based commerce has seen a change in the types of data firms report to external shareholders. More than ever before, companies are discussing and disclosing data on the number of customers acquired and lost, customer lifetime value, and more. This has fueled an increasing interest in linking the value of a firm's customers to the overall value of the firm, with the term customer-based corporate valuation being using to describe such efforts. While a number of researchers in the fields of marketing and accounting have explored this idea, their underlying models of customer acquisition and retention do not adequately reflect the empirical realities associated with these behaviors, and the associated valuation models do not live up to the standards of finance professionals. We develop a framework for valuing subscription-based firms that addresses both issues, and apply it to data from Dish Network and Sirius XM Holdings.

Keywords: customer lifetime value; customer equity; shareholder value; valuation.

1 Introduction

The relevance and popularity of subscription-based businesses — businesses whose customers pay a periodically recurring fee for access to a product or service — has grown considerably in recent years. Previously dominated by newspapers, magazines, and telecommunications companies, the subscription-based business model has made strong inroads into consumer software (Microsoft 365), food preparation (Blue Apron), health and beauty products (Dollar Shave Club), and a large array of subscription-based software-as-a-service (SaaS) enterprises in the B2B space, as businesses look to increase the predictability of their revenue streams. Many experts have written in depth about this topic (Baxter 2015; Janzer 2015; Warrillow 2015).

The increased popularity of subscription-based businesses has brought with it an increase in the public disclosure of data on (but not limited to) customer churn, customer/subscriber acquisition costs, average revenue per user, and customer lifetime value (CLV). The price of a company's stock reflects and incorporates investors' beliefs regarding the future cash flows the company will generate. For the subscription-based businesses, the primary source of future cash flows is customers. Therefore, customer data is important to investors and is being used by analysts as they make their recommendations. For example, a class-action lawsuit was taken out again Netflix in response to changes in its reporting of such data (SCAC 2004). Analyst reports from Thomas Weisel Partners, Vintage Research, First Albany Capital and Delafield Hambrecht, made public as part of the litigation, all strongly emphasize customer data in general (and the size of the total subscriber base over time in particular) when justifying their investment recommendations.

The pioneering work of Gupta, Lehmann, and Stuart (2004) was the first to explicitly link firm value to CLV for public companies. However, their treatment of the valuation problem suffers from two major issues. First, their CLV calculations are performed assuming a constant retention rate, which can result in an undervaluing of existing customers (Fader and Hardie 2010). Second, their valuation framework does not incorporate key financial/accounting issues such as firm capital structure and non-operating assets. While other researchers, most notably

Schulze, Skiera, and Wiesel (2012), have built upon this seminal work, the underlying models of customer behavior and the associated valuation frameworks are not up to the standards expected by marketers and financial professionals, respectively.

Our objective is to present a framework for valuing subscription-based business. The parameters of the underlying model of customer behavior can be estimated using only publicly disclosed customer data, making it suitable for passive investors valuing a going concern. We present models of the firm's acquisition and retention processes that accommodate factors such as customer heterogeneity, duration dependence, seasonality, and changes in population size. We explicitly account for the fact that publicly reported data are typically aggregated (temporally and across customers) and suffer from missingness (i.e., the reported data are not available for all periods).

The paper is organized as follows. In the next section, we discuss the principles of customer-based corporate valuation, first reviewing the basic concepts of firm valuation and then exploring the nature of the customer data typically released by subscription-based businesses. Following a review of the literature, we present our model of customer behavior for such settings, presenting models for customer acquisition, retention, and spend. We then provide an empirical analysis that explores how such a model can be fit to real public company data; the two firms considered in our analysis are Dish Network and Sirius XM. After demonstrating the validity of our model, we present our valuations of the firms, and explore other insights that can be derived using our model. We conclude with a discussion of the results and future work.

2 The Logic of Customer-Based Corporate Valuation

Before reviewing the literature and then developing our framework for valuing subscription-based business using publicly disclosed data, let us first review a standard approach to firm valuation (identifying the key information requirements) and then identify the data that are typically available inside the firm (in contrast to the data that firms with a subscription-based business model tend to report to the public).

2.1 Valuation 101

According to standard corporate valuation theory (Damodaran 2012; Greenwald et al. 2004; Holthausen and Zmijewski 2014; Koller, Goedhart, and Wessels 2015), the value of a firm equals the value of the operating assets (OA) plus the non-operating assets (NOA), minus the net debt (ND) of the firm. Denoting the value of the firm at time T by SHV_T (for shareholder value),[1] we have

$$\mathrm{SHV}_T = \mathrm{OA}_T + \mathrm{NOA}_T - \mathrm{ND}_T\,. \tag{1}$$

The value of a firm's operating assets (OA) is equal to the sum of all future free cash flows (FCFs) the firm will generate,[2] discounted at the weighted average cost of capital (WACC):

$$\mathrm{OA}_T = \sum_{t=0}^{\infty} \frac{\mathrm{FCF}_{T+t}}{(1+\mathrm{WACC})^t}. \tag{2}$$

FCF is equal to the net operating profit after taxes (NOPAT) minus the difference between capital expenditures (CAPEX) and depreciation and amortization (D&A), minus the change in non-financial working capital (ΔNFWC):

$$\mathrm{FCF}_t = \mathrm{NOPAT}_t - \big(\mathrm{CAPEX}_t - \mathrm{D\&A}_t\big) - \Delta\mathrm{NFWC}_t. \tag{3}$$

The most important ingredient of FCF is NOPAT, which is a measure of the underlying profitability of the operating assets of the firm. NOPAT is equal to revenues (REV) times the con-

[1] In this work, we take the perspective of a passive investor valuing a going concern. This same basic valuation framework can also be used by owners and managers who can influence the firm's decision making to evaluate alternative investment decisions (Damodaran 2012; Koller, Goedhart, and Wessels 2015). The idea of value-based management (i.e., the notion that maximizing shareholder value should be the guiding principle when making strategic decisions) was popularized by Rappaport (1986), and has influenced thinking in the marketing strategy literature (e.g., Day and Fahey 1988; Doyle 2000; Srivastava, Shervani, and Fahey 1997, 1998). As such, most of the SHV-related discussions in the marketing literature relate to strategic decision making, as opposed to firm valuation.

[2] Strictly speaking, we are referring to *expected* free cash flows.

tribution margin ratio $(1 - \text{VC})$ minus fixed operating costs (FC), after taxes (where TR is the corporate tax rate for the firm):

$$\text{NOPAT}_t = (\text{REV}_t \times (1 - \text{VC}_t) - \text{FC}_t) \times (1 - \text{TR}_t)\,. \tag{4}$$

The other elements of Equation 3 make adjustments for balance sheet-related cash flow effects, and are generally of secondary importance to the value of the firm.

The framework summarized above is a discounted cash flow (DCF) model, which is the de-facto industry standard way in which operating assets are valued within the financial community.[3] At the heart of any such valuation exercise is the estimation of period-by-period FCF, central to which are estimates of period-by-period revenue (Equations 3 and 4). The task of generating accurate revenue projections has received surprisingly little attention in the finance community (Damodaran 2005).

2.2 A Data Structure for Subscription-Based Businesses

Let us assume that the firm has a monthly internal reporting period. The key numbers of interest are monthly revenues, which we denote by $R(m)$ (where $m = 1$ corresponds to the firm's first month of commercial operations).

While some researchers would be tempted to approach the task of forecasting revenue by using a time-series model, a few moments of reflection will suggest that it makes more sense to first decompose these aggregate revenue numbers, separately model the constituent components, and then combine the forecasts of these components to arrive at the desired revenue forecasts.[4] First, we should recognize that revenue comes from customers, so decomposing

[3] As Damodaran (2012, p. 925) notes, "[t]he problem in valuation is not that there are not enough models to value an asset, it is that there are too many. Choosing the right model to use in valuation is as critical to arriving at a reasonable value as understanding how to use the model." Using the decision framework developed in Damodaran (2012, Chapter 34), it is clear that the discounted cash flow approach used in this paper is the most appropriate for the task at hand.

[4] There is an obvious parallel with the new product sales forecasting literature (e.g., Fourt and Woodlock 1960; Eskin 1973; Fader, Hardie, and Huang 2004), where we first decompose total sales into its trial and repeat compo-

revenue into its "number of customers" and "average revenue per customer" components would be a good start. Second, as we think about the number of customers the firm has in a month, it makes sense to decompose this quantity into the number of new customers acquired that month and the number of customers acquired in previous months who still have a relationship with the firm. Knowing the number of new customers acquired of each month is a critical input to any valuation exercise, especially for firms with high subscriber acquisition costs. It is important to note that such information will be overlooked if we simply apply a time-series model to the revenue numbers.

As a starting point, let us think about what lies behind the "total number of customers" number. It is helpful to think of a "number of customers" matrix, $C(\cdot,\cdot)$, which tracks customer behavior by time of acquisition. With reference to Figure 1 (where the columns correspond to (calender) time since the start of the firm's commercial operations and the rows correspond to acquisition cohorts), let $C(m, m')$ be the number of customers acquired by the firm in month m who are still active in month m'. It follows that the total number of customers the firm has at the end of month m' is given by the column total $C(., m') = \sum_{m=1}^{m'} C(m, m')$. The number of customers in any cohort must be non-increasing over time (i.e., $C(m, m') \geq C(m, m'')$ for $m' < m''$).

The $C(\cdot,\cdot)$ matrix, along with $R(\cdot)$, lies that the heart of a number of customer metrics reported both internally and externally.

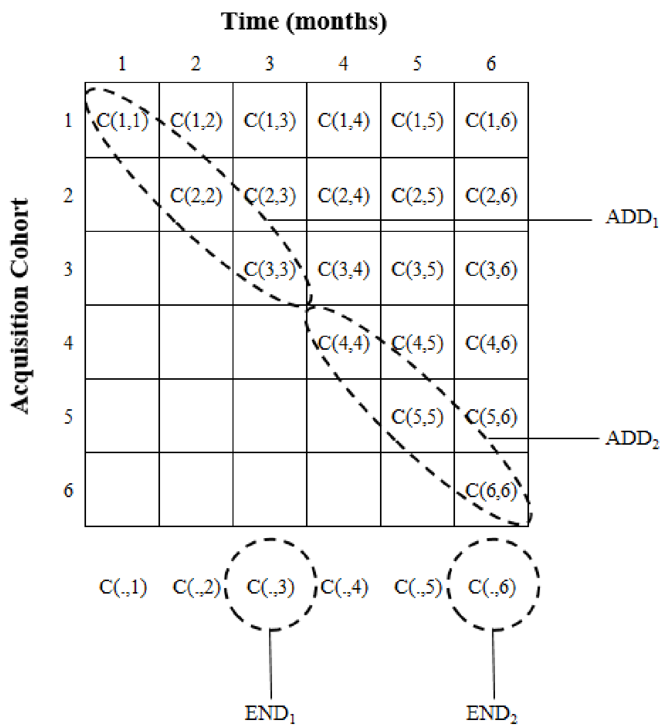
- A sophisticated subscription-based firm will report the $C(\cdot,\cdot)$ matrix internally, either in its raw form or as cohort-by-cohort survival percentages ($C(m, m')/C(m, m) \times 100\%$) (e.g., Martínez-Jerez et al. 2013).

- The number of customers acquired each month by the firm is given by

$$A(m) = C(m, m)\,. \tag{5}$$

nents, ii) develop separate models for trial and repeat sales, and ii) then combine the associated forecasts of these components to arrive at a forecast of total sales.

Figure 1: The "Number of Customers" Matrix



- The number of customers "lost" each month by the firm is given by

$$L(m) = \begin{cases} 0 & m = 1 \\ C(.,m-1) - [C(.,m) - C(m,m)] & m = 2,3,4,\ldots \end{cases} \tag{6}$$

  (It follows that an aggregate monthly churn rate can be computed as $L(m)/C(.,m-1)$.)

- For most firms with a subscription-based business model, the average revenue per subscriber is relatively constant across customers during a given period of time.[5] Let us denote this quantity by $ARPU(m)$ and compute it in the following manner:

$$ARPU(m) = R(m) \Big/ \frac{C(.,m-1) + C(.,m)}{2}. \tag{7}$$

[5]In contrast, average revenue per subscriber for firms with a non-subscription-based business model tends to vary considerably across customers in any given period of time.

(The denominator is the average number of customers the firm has during month m.)

Publicly disclosed customer data are typically reported quarterly with the associated unit of time being the quarter; as such, they represent a temporal aggregation of the true underlying process. Commonly reported measures include the number of customers active at the end of each quarter (END_q), and the number of customers added and lost each quarter (ADD_q and LOSS_q, respectively). Assuming the firm started operations at the beginning of a reporting quarter (i.e., $q = 1$ comprises $m = \{1, 2, 3\}$; equivalently, the first month of each quarter is either January, April, July or October),

$$\text{END}_q = C(.,3q) \tag{8}$$

$$\text{ADD}_q = A(3q-2) + A(3q-1) + A(3q) \tag{9}$$

$$\text{LOSS}_q = L(3q-2) + L(3q-1) + L(3q) \tag{10}$$

This mapping from the internal "number of customers" matrix to ADD and END is illustrated in Figure 1.

Quarterly revenues (REV_q) are given by

$$\text{REV}_q = R(3q-2) + R(3q-1) + R(3q)\,. \tag{11}$$

The challenge we face is how to make projections of $R(m)$ and $A(m)$ far into the future (as required for calculating for the FCF numbers) using the publicly reported ADD, LOSS, END, and REV numbers. We pursue this important task in our Model Development section below but first we discuss how other researchers have utilized the valuation concepts and data structures discussed here.

3 Literature Review

The idea of value-based management (i.e., the notion that maximizing shareholder value should be the guiding principle when making strategic decisions) gained popularity in the 1980s, and the associated writings — especially Rappaport (1986) — brought the basic principles of firm valuation (as reviewed in Section 2.1) to a broader, non-finance audience.

In his review of the valuation literature, Damodaran (2005, p. 1) writes: "Given the centrality of its role, you would think that the question of how best to value a business, private or public, would have been well researched. [...] [T]he research into valuation models and metrics in finance is surprisingly spotty, with some aspects of valuation, such as risk assessment, being deeply analyzed and others, such as how best to estimate cash flows ... not receiving the attention that they deserve."

Kim, Mahajan, and Srivastava (1995) were the first marketing academics to recognize the potential for using some of the models of customer behavior developed by marketing scientists to generate the key inputs for estimating cash flows. They used the logistic internal-influence model for the diffusion of an innovation (which is equivalent to Fisher and Pry's (1971) model of technology substitution) to characterize (and then project) the market penetration of mobile phones (and therefore the associated revenues of a cellular communication company), resulting in an estimate of the market value of a business explicitly based on a model of customer behavior.[6] Pioneering as it was, the biggest shortcoming in their analysis was that they did not consider the reality of customer churn (i.e., it is assumed that once the customer has adopted the service, they remain as a customer forever).

Driven in part by the interest in moving from transaction-oriented/product-centric marketing strategies to relationship-oriented/customer-centric marketing strategies (with their emphasis on customer acquisition, retention, and development), the 1990s saw the notion of customer lifetime value (CLV) — defined as "the present value of the future cash flows attributed to the

[6]It is important to note that they applied the model at the level of the industry, not the firm, in their empirical analysis.

customer relationship" (Pfeifer, Haskins, and Conroy 2005, p. 17) — emerge from the confines of specialized direct/database marketing firms and become what is now a fundamental concept for most marketers. Blattberg and Deighton (1996) introduced the concept of "customer equity" (CE), which is the sum of the lifetime values of the firm's customers, both current and future. Kumar and Shah (2015) provide a comprehensive guide to the literature on customer equity.

The pioneering work of Gupta, Lehmann, and Stuart (2004) (hereafter, GLS) was the first to explicitly link CLV and firm value. Underpinning their work was the logistic internal-influence model to characterize customer acquisitions and a simple model for the CLV of acquired customers. After calibrating the models using publicly available data (along with expert judgment), they arrived at estimates of market value for five listed companies. However, their treatment of the valuation problem suffers from two major issues. First, their CLV calculations are performed assuming a constant retention rate. Second, their valuation framework does not incorporate key financial/accounting issues such as firm capital structure and non-operating assets.

A number of researchers have built on GLS's seminal work. Most notably, Schulze, Skiera, and Wiesel (2012) (hereafter SSW) provide a thorough treatment of how CE relates to firm value using financial valuation theory, addressing many of the financal/accounting issues associated with the valuation aspect of GLS's work. Several researchers have explored a number of technical issues associated with any valuation exercise. Pfeifer (2011) shows how one must be careful with the timing of cash flows when estimating retention rates and the value of the firm's existing customers (which he calls current-customer equity, CCE) using publicly disclosed company data. However, he stops short of providing his own estimate of CCE for any firm, let alone an estimate of the value of the firm as a whole. Fader and Hardie (2010) show how assuming a constant retention rate (i.e., ignoring the phenomenon of increasing retention rates at the level of the cohort) results in downward-biased estimates of the future or *residual* lifetime value (RLV) of the firm's current customers. Other key papers include Kumar and Shah (2009), Libai, Muller, and Peres (2009), and Wiesel, Skiera, and Villanueva (2008).

These ideas have been gaining attention and respect outside of marketing. Within the accounting literature, for instance, Bonacchi, Kolev, and Lev (2015) provide a systematic analysis across multiple companies linking CCE to shareholder value. Other accounting-related work includes Andon, Baxter, and Bradley (2001), Bonacchi, Ferrari, and Pellegrini (2008), Bonacchi and Perego (2012), Boyce (2000), Gourio and Rudanko (2014), and Hand (2015).

We summarize the literature and our contributions to it in Table 1. Since we will use the models proposed by GLS and SSW as benchmarks in our empirical analysis, let us briefly comment on the structure of these models. Both rely on the logistic internal-influence model to model customer acquisition and both assume a homogeneous retention rate in their CLV calculations. The impacts of seasonality and macroeconomic conditions have not been incorporated into their models, nor have the issues of (temporal) aggregation and missing data associated with the information released by companies. These are issues that we will address in the valuation framework we develop below.

While many of the papers reviewed above discuss DCF methods for firm valuation (often anchoring on Rappaport's (1986) expression for SHV), they do not explicitly make use of such a framework when generating an estimate of firm value. Rather, they take what Skiera and Schule (2014) call a customer-based valuation approach. Skiera and Schulze (2014, p. 123) first state that "[c]ustomer-based valuation first uses information about the customer base (for example, number of customers, contribution margin per customer, retention rate) to determine the value of the firm," and then describe an approach based on the (residual) lifetime value of existing customers and the lifetime value of as-yet-to-be-acquired customers, multiplying these two quantities by the number of current and expected future customers (respectively) and adjusting for various financial considerations. Their position is that DCF and "customer-based valuation" methods are fundamentally different.

We believe that this is more polarizing than it needs to be. With reference back to Figure 1, previous "customer-based valuation" methods are performing on a row-by-row basis what are effectively NPV calculations across columns and then summing up across rows. "DCF methods" (as outlined in Section 2.1) are summing up the columns and then effectively performing

Table 1: Literature Overview

	Setting		Customer Dynamics			Valuation Elements				
Paper	*Data Source*	*Missing Data (left censoring)*	*Heterogeneity (churn)*	*Duration Dependence*	*Covariates*	*DCF Model*	*WACC Estimated*	*Debt / Other Assets*	*Customers Valued*	*Rolling Validation*
Kim, Mahajan, and Srivastava (1995)	Public + Experts	No	No	No	No	Yes	No	No	Current + Future	No
Gupta, Lehmann, and Stuart (2004)	Public + Experts	No	No	No	No	No	No	No	Current + Future	Yes
Wiesel, Skiera, and Villanueva (2008)	Public	No	No	No	No	No	No	No	Current	No
Kumar and Shah (2009)	Private	No	No	No	Yes	No	No	No	Current	Yes
Libai, Muller, and Peres (2009)	Public + Experts	No	No	No	Yes	No	No	No	Current + Future	Yes
Fader and Hardie (2010)	None	No	Yes	No	No	No	No	No	Current	No
Pfeifer (2011)	None	No	No	No	No	No	No	No	Current	No
Schulze, Skiera, and Wiesel (2012)	Public	No	No	No	No	No	Yes	Yes	Current + Future	Yes
Bonacchi, Kolev, and Lev (2015)	Public	No	No	No	No	No	Yes	No	Current	No
Proposed	Public	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Current + Future	Yes

an NPV calculation across these column totals. Given the same customer matrix $C(\cdot,\cdot)$, and assuming all the various accounting issues are handled correctly, both approaches should yield the same estimate of firm value.

While marketers may naturally be comfortable with discussions of firm valuation that are explicitly based on CLV and CE calculations, finance professionals (including the CFO and company directors) are more comfortable with a valuation model based on estimates of period-by-period FCF. What marketers can bring to the table are customer-based methods for estimating these period-by-period cash flows. As developed in Section 2.2 above, we feel that the $C(\cdot,\cdot)$ matrix lies at the heart of any such effort, and we now turn our attention to the development of a model that can be calibrated on publicly available data used to project $C(\cdot,\cdot)$ far into the future, thereby giving us the required inputs for computing future period-by-period cash flow.

4 Model Development

Our goal is to develop a model of customer behavior that can be used to generate long-run projections of $R(m)$ and $A(m)$, one whose parameters can be estimated using only publicly reported ADD, LOSS, END, and REV numbers. It is important that our approach to parameter estimation accounts for the "missingness" and aggregation associated with the data reported by companies. This "missingness" typically takes on one of two forms. First, there is the issue of left-censoring. For example, while Sirius XM (one of the companies considered in our empirical analysis) began commercial operations in 2001/2002,[7] it started disclosing paying customer data in Q3 2008. Second, there is the issue that some measures are reported for some period of time (e.g., END) before being complemented by other measures (e.g., ADD and LOSS). With respect to aggregation, the publicly disclosed customer data is typically reported quarterly with the associated unit of time being the quarter, whereas the firm is operating on a

[7] The two companies that later merged to form Sirius XM started commercial operations in September 2001 and February 2002.

finer time interval (which, for the purposes of our analysis, we assume to be the month).[8]

At the heart of this work are models for the customer acquisition and retention processes that allow us to project $C(\cdot,\cdot)$ into the future. Coupled with a model for $ARPU(m)$, we can then generate our projections of $R(m)$.

We start by describing our model for the retention process. This assumes we know how many customers the firm acquires each month. We then describe our model for the customer acquisition process, one that takes into consideration the possibility of reacquiring customers who have previously churned, and then examine how to jointly estimate the parameters of these two models. Finally, we present a simple model for the evolution of $ARPU(m)$, and then outline how to bring together all of these submodels to generate the desired projections of $R(m)$.

4.1 The Retention Process

Let the survival function $S_R(m'-m|m)$ denote the probability that a customer acquired in month m remains an customer for at least $m'-m$ months. Having acquired $A(m) = C(m,m)$ customers in month m, it follows that

$$C(m,m') = C(m,m) \times S_R(m'-m|m)\,, m' \geq m\,. \tag{12}$$

Our objective is to specify an accurate yet parsimonious survival model for the duration of customers' relationships with the firm. In addition to capturing the effects of (cross-sectional) heterogeneity and duration dependence, we want to accommodate time-varying covariates to control for the effects of seasonality and macroeconomic conditions.

We use a proportional hazards model with a Weibull baseline, and capture cross-sectional heterogeneity in the baseline churn propensity using a gamma distribution. This is a well-accepted model for duration data and has been proven to be quite effective and robust in a

[8]Our model can also be used in situations where customer data is only reported annually with minimal modification.

number of different application settings (Morrison and Schmittlein 1980; Moe and Fader 2002; Schweidel, Fader, and Bradlow 2008b).

Given a customer's individual-specific baseline propensity to churn (λ_R), a homogeneous retention process shape parameter (c_R), time-varying retention covariates ($\mathbf{X}_R(m, m') = [\mathbf{x}_R(m),\ \mathbf{x}_R(m+1),\ \ldots,\ \mathbf{x}_R(m')]$), and the coefficients associated with the retention covariates ($\boldsymbol{\beta}_R$),

$$S_R(m'-m|m, \mathbf{X}_R(m+1, m'); \lambda_R, c_R, \boldsymbol{\beta}_R) = \exp(-\lambda_R B_R(m, m'))\,,$$

where

$$B_R(m, m') = \sum_{i=m+1}^{m'} [(i-m)^{c_R} - (i-m-1)^{c_R}] e^{\boldsymbol{\beta}_R \mathbf{x}_R(i)}. \tag{13}$$

Following Schweidel, Fader, and Bradlow (2008b) and Jamal and Bucklin (2006), we expect $c_R \geq 1$. When $c_R = 1$, it reduces to an exponential baseline proportional hazards model.

Assuming λ_R is distributed gamma(r_R, α_R) across the population, the unconditional probability that a customer acquired in month m survives at least $m' - m$ months is

$$\begin{aligned} &S_R(m'-m|m, \mathbf{X}_R(m+1, m'); r_R, \alpha_R, c_R, \boldsymbol{\beta}_R) \\ &\qquad = \int_0^\infty S(m'-m|m, \mathbf{X}_R(m+1, m'); \lambda_R, c_R, \boldsymbol{\beta}_R) f(\lambda_R|r_R, \alpha_R) d\lambda_R \\ &\qquad = \left(\frac{\alpha_R}{\alpha_R + B_R(m, m'-m)}\right)^{r_R}. \end{aligned} \tag{14}$$

Plugging this survival function into Equation 12 allows us to predict the number of active customers in future months for the month m cohort. Therefore, if we know $C(m, m)$ over all months, we can predict the remainder of the upper triangular matrix in Figure 1 as well as the number of customers lost each month (Equation 6). Estimates of LOSS_q and END_q follow from Equations 9 and 10 for all $q = 1, 2, \ldots$.

In theory, we could estimate the model parameters $(r_R, \alpha_R, c_R, \boldsymbol{\beta}_R)$ by minimizing the sum of squared differences between our model-based estimates of LOSS_q and the reported numbers. However this assumes we know the monthly customer acquisition numbers, $A(m)$, which is not the case. We only have quarterly customer additions ADD_q and some of these observations are probably missing. We therefore need to develop a model of the acquisition process whose parameters can be estimated using the reported ADD_q data. These two models will be estimated simultaneously to give us the required $A(m)$ and $L(m)$ numbers.

4.2 The Acquisition Process

At first glance, specifying a model for the acquisition of customers over time seems to be a relatively simple exercise. The Bass model (1969) or a simplified variant such as the logistic internal-influence model (as used by GLS and SSW) would appear to be the obvious choice. However, for the following four reasons, this is not the case:

1. It assumes that all churning customers disappear forever — once an acquired customer has churned, he/she cannot re-enter the pool of 'potential adopters.' SSW attempt to overcome this problem by using the logistic internal-influence model to characterize the number of net total customers (i.e., the number of customers after churn).[9]

2. It assumes the population size is fixed, when we know that the number of potential customers is typically increasing over time due to population growth.

3. The Bass model and its simplified variants have a number of unfavorable properties, most notably the fact that the resulting adoption curve is symmetric about the period of peak acquisition. In real datasets, skewness about the peak is almost always present.

4. It ignores the effects of seasonality and macro-economic events.

We therefore develop a model from first principles that addresses each of these issues.

Let $POP(m)$ denote the size of the population in month m, with $POP(0)$ being the population size when the firm first commences operations. We assume $POP(m)$ is non-decreasing over time.

[9] Libai, Muller, and Peres (2009) extend the basic Bass model to allow for lost customers re-entering the pool of potential adopters, but they assume a constant retention rate.

Each month sees the formation of a new prospect pool of size $M(m)$. We set $M(0)$, the size of firm's prospect pool when it commences operations, to the size of the population at that time. The size of the prospect pool in the company's second month of operation is simply the increase in the size of the population over the preceding month. Thereafter, the size of the prospect pool is equal to the growth in the population during the preceding month, plus the number of customers who churned in the previous month:

$$M(m) = \begin{cases} POP(m) & m = 0 \\ POP(m) - POP(m-1) + L(m-1) & m = 1, 2, 3, ... \end{cases} \tag{15}$$

We assume that population growth is the only source of potential adopter growth over time, aside from previously churned customers.

Once a prospect pool has formed, some time will elapse before individuals within that pool are acquired as customers.[10] Let $F_A(m' - m|m)$ denote the probability that a member of prospect pool m is aquired by the end of month m'. It follows that the total number of new customers in month m is

$$A(m) = \sum_{i=0}^{m-1} M(i) \times \big[F_A(m-i|i) - F_A(m-i-1|i)\big]. \tag{16}$$

We model the time it takes for a prospect to become a customer using a split-hazard model. A proportion π_{NA} of each prospect pool will never be acquired. For those that are potential customers, we characterize the time to acquisition using a proportional hazards model with a Weibull baseline, and capture cross-sectional heterogeneity in the baseline acquisition propensity using a gamma distribution.

Given a prospect's individual-specific baseline propensity to be acquired (λ_A), a homogeneous acquisition shape parameter (c_A), time-varying acquisition covariates ($\mathbf{X}_A(m+1, m') =$

[10] In line with most of the work on modeling the diffusion of innovations, we ignore the intermediate role of awareness as we do not have sufficient data to account for it.

$[\mathbf{x}_A(m+1),\ \mathbf{x}_A(m+2),\ \ldots,\ \mathbf{x}_A(m')]$), and the coefficients associated with the acquisition covariates ($\boldsymbol{\beta}_A$),

$$F_A(m'-m|m, \mathbf{X}_A(m+1, m'); \lambda_A, c_A, \pi_{NA}, \boldsymbol{\beta}_A) = (1-\pi_{NA})\big[1-\exp(-\lambda_A B_A(m, m'))\big], \quad (17)$$

where

$$B_A(m, m') = \sum_{i=m+1}^{m'} [(i-m)^{c_A} - (i-m-1)^{c_A}] e^{\boldsymbol{\beta}_A \mathbf{x}_A(i)}. \quad (18)$$

Assuming λ_A is distributed gamma(r_A, α_A) across the population distribution, the unconditional probability that a customer from prospect pool m will be acquired by the end of month m' is

$$\begin{aligned} F_A(m'-m|m, \mathbf{X}_A(m+1, m'); r_A, \alpha_A, c_A, \pi_{NA}, \boldsymbol{\beta}_A) \\ &= \int_0^\infty F_A(m'-m|m, \mathbf{X}_A(m+1, m'); \lambda_A, c_A, \pi_{NA}, \boldsymbol{\beta}_A) f(\lambda_A|r_A, \alpha_A) d\lambda_A \\ &= (1-\pi_{NA})\left[1 - \left(\frac{\alpha_A}{\alpha_A + B_A(m, m')}\right)^{r_A}\right]. \end{aligned} \quad (19)$$

This acquisition model is flexible yet parsimonious. Parsimony is especially important in limited data settings (such as those considered here) because, as shown in Van den Bulte and Lilien (1997), ill-conditioning is a serious enough problem with small sample sizes that adding new predictors to alleviate model misspecification concerns may make the resulting model fit (and forecast) worse than it had been prior to the introduction of those covariates.

4.3 Parameter Estimation for the Acquisition and Retention Processes

We estimate the parameters of the acquisition and retention process models jointly using non-linear least squares, minimizing the sum of squared differences between the actual and model-based estimates of quarterly acquisitions and losses. Let $\boldsymbol{\psi}$ denote the acquisition and retention process model parameters collectively, $\boldsymbol{\psi} \equiv (r_A, \alpha_A, c_A, \pi_{NA}, \boldsymbol{\beta}_A, r_R, \alpha_R, c_R, \boldsymbol{\beta}_R)$, and let Q be the number of quarters from the commencement of the firm's commercial operations to the end of the model calibration period.

If the firm reports the quarterly numbers from the very start of its operations, our parameters are those that minimize the following sum-of-squared errors:

$$\mathrm{SSE}_{FULL} = \sum_{q=1}^{Q} \left\{ \left(\mathrm{ADD}_q - \widehat{\mathrm{ADD}}_q\right)^2 + \left(\mathrm{LOSS}_q - \widehat{\mathrm{LOSS}}_q\right)^2 \right\} + \left(\mathrm{END}_Q - \widehat{\mathrm{END}}_Q\right)^2, \quad (20)$$

where $\widehat{\mathrm{ADD}}_q$, $\widehat{\mathrm{LOSS}}_q$, and $\widehat{\mathrm{END}}_q$ are the model-based estimates of these quantities computed using $\hat{\boldsymbol{\psi}}$.[11] Note that we optimize over all parameters jointly because of the dependence of the retention process on the acquisition process (i.e., customers cannot churn until they have been acquired) and vice versa (i.e., churning customers enter future prospect pools).

Equation 20 assumes that there is no missing data. However, this is rarely, if ever, the case. Most companies start disclosing ending customer count data (END_q) some number of quarters into the company's operations (call it q_A), then begin disclosing customer ADD and LOSS data in a later quarter (call it q_B, where $q_B \geq q_A$). In such cases, our parameters are those that minimize the following sum-of-squared errors:

$$\begin{aligned}\mathrm{SSE}_{MISS} = &\sum_{q=q_A+1}^{q_B} \left[\left(\mathrm{END}_q - \mathrm{END}_{q-1}\right) - \left(\widehat{\mathrm{END}}_q - \widehat{\mathrm{END}}_{q-1}\right)\right]^2 \\ &+ \sum_{q=q_B+1}^{Q} \left\{ \left(\mathrm{ADD}_q - \widehat{\mathrm{ADD}}_q\right)^2 + \left(\mathrm{LOSS}_q - \widehat{\mathrm{LOSS}}_q\right)^2 \right\} \\ &+ \left(\mathrm{END}_Q - \widehat{\mathrm{END}}_Q\right)^2, \end{aligned} \quad (21)$$

This accounts for the shortened contiguous customer addition and loss data, and the missingness present at the beginning of the time series.

[11] This assumes the firm started operation at the beginning of a reporting quarter, as discussed in Section 2.2. If this is not the case, minor modifications to Equations 20 and 21 are needed.

4.4 Average Revenue Per User

We make use of a simple time-series model to capture (and project) the evolution of $ARPU(m)$. Assuming linear growth in ARPU,[12] we can use a simple time-trend regression:

$$ARPU(m) = \beta_0 + \beta_1 m + \epsilon(m), \quad \epsilon(m) \sim \mathcal{N}(0, \sigma^2). \tag{22}$$

The mean of many economic and financial time series is non-stationary (Zivot and Wang 2006). When this is so, the fitted residuals of the regression given in Equation 22 will fail tests for non-stationarity, the most popular of which is the Augmented Dickey-Fuller test (Dickey and Fuller 1979; Elliott, Rothenberg, and Stock 1996). If this is the case, the parameter estimates from Equation 22 are invalid, and we should instead use an ARIMA(0,1,0) model:

$$ARPU(m) = ARPU(m-1) + \beta_0 + \epsilon(m), \quad \epsilon(m) \sim \mathcal{N}(0, \sigma^2). \tag{23}$$

The parameters of either model are estimated using maximum likelihood[13].

$ARPU(m)$ is a standard internally reported measure for a subscription-based firm. Some firms do report quarterly ARPU publicly, but this data cannot be used in general because there are no well-accepted standards for calculating it. As Dish stated in its 2014 annual filing, "We are not aware of any uniform standards for calculating ARPU and believe presentations of ARPU may not be calculated consistently by other companies in the same or similar businesses." As there is no standard definition of ARPU, different firms may have different definitions for it, picking and choosing what sources of revenue to include in the numerator. As such, the reported ARPU numbers may not be representative of all revenue derived from the customer base.

Revenue numbers are more reliable. However, they are only provided quarterly, so we need to impute monthly revenues. For $m \in \{3q-2, 3q-1, 3q\}$, the revenue in month m is equal to

[12] If we assume that ARPU grows at a constant growth rate over time, we would use $\log\big(ARPU(m)\big) = \beta_0 + \beta_1 m + \epsilon(m)$.

[13] As in GLS and SSW, we assume that spend and churn are uncorrelated. The data is too limited to identify such a correlation, and the lack of heterogeneity in spend limits the practical benefit of allowing for it.

the customer-weighted share of total revenue in quarter q:

$$R(m) = \frac{C(.,m-1) + C(.,m)}{C(.,3q-3) + 2C(.,3q-2) + 2C(.,3q-1) + C(.,3q)} \times \text{REV}_q\,. \tag{24}$$

(Strictly speaking, we are using $\widehat{C}(\cdot,\cdot)$, computed using the estimated parameters of the acquisition and retention processes.) Having imputed $R(m)$, our estimates of $ARPU(m)$ follow from Equation 7.

4.5 Bringing It All Together

Taking a step back, recall that our goal since Section 2.2 has been to generate long-term projections of $R(m)$ and $A(m)$, from which we can compute estimates of period-by-period FCF and then the value of the firm. We now outline the process by which we compute these revenue numbers using the models described above.

1. We estimate the parameters of the acquisition and retention processes (Section 4.3). Assuming the firm has been in operation for Q quarters, we then compute our estimate of the $3Q \times 3Q$ matrix $C(\cdot,\cdot)$, the diagonal of which is our estimate of the number of customers acquired each month over this time period, and the rows of which are estimates of the number of customers in each cohort that survive each of the subsequent months.

2. As outlined in Section 4.4, we use $\widehat{C(\cdot,\cdot)}$ and the reported quarterly revenue numbers to impute the corresponding monthly revenue numbers, from which we estimate the parameters of our model for average revenue per user.

3. In order to project $C(\cdot,\cdot)$ into the future, we need estimates of $POP(m)$ over the time horizon of interest. In some cases, such data may be available from a secondary source. In the absence of such a source, we can use a simple model for forecasting $POP(m)$. For example, we could use the long-run compound growth rate in $POP(m)$ and assume it holds going into the future.

4. Having projected $C(\cdot,\cdot)$ far into the future (i.e., to a point in time where the present value of any associated cash flows is effectively zero), we compute the column totals $C(.,m)$ to give us the total number of customers for each month.

5. We compute expected $ARPU(m)$ across this time horizon using Equation 22 or 23. Rearranging Equation 7, it follows that

$$R(m) = ARPU(m) \times \frac{C(.,m-1) + C(.,m)}{2}\,. \tag{25}$$

With the revenues estimated, the remainder of our valuation model is for all intents and purposes the same as what a financial professional would do when building a DCF valuation model. In the next section, we bring this valuation model to life from start to finish using data for two public companies.

5 Empirical Analyses

We first apply our model of customer behavior to data from Dish Network Corporation (Nasdaq: DISH), a large pay-TV service provider. We estimate the parameters of the model, evaluate its in-sample fit, evaluate the predictive validity of the model by performing rolling two-year-ahead forecasts over all possible calibration periods, and compare its performance to that of the models of customer behavior proposed by GLS and SSW. After demonstrating the validity of the model, we then use its revenue projections (along with the associated estimates of customer acquisition) to arrive at our estimate of the value of Dish's shareholder's equity. Next, to further establish the robustness of our proposed model, we apply it to a second publicly-traded company, Sirius XM Holdings (Nasdaq: SIRI), a satellite radio service provider. We conclude by exploring some other insights into customer behavior that can be derived using the model.

5.1 Dish Network

Dish commenced operations in March 1996 (DISH 2015),[14] and end-of-period customer counts were first disclosed that quarter. However the gross customer acquisition data is left censored — the first time that gross customer additions were disclosed was seven quarters later, in Q1 1998 (i.e., with reference to Equation 21, $q_A = 0$ while $q_B = 7$). All historical customer data (ADD_q, LOSS_q, END_q and REV_q) come from Dish's quarterly and annual reports, Forms 10-Q and 10-K, respectively. We model this customer data up to and including Q1 2015 (i.e., $Q = 77$). The vast majority of Dish's revenues come from its subscriber base.[15]

We use the same four time-varying covariates in our models of the acquisition and retention processes: three quarterly dummy variables to capture seasonal fluctuations in the propensity to sign-up to and churn from the service, and a "Great Recession" dummy variable to account for the diminished propensity to sign-up and the increased propensity to churn during that recession.[16] Given the nature of the Dish's service offering, our unit of population is the household. We use data on US household growth provided in the US Census Bureau's CPS/HVS data tables.

5.1.1 Parameter Estimates and Evaluation of Fit

We first estimate the parameters of the acquisition and retention models using all the available data. The parameters are reported in Table 2; the associated model SSE is $310{,}821$. The story told by these parameters is consistent with what Dish has disclosed in its public filings. With reference to the coefficients of the quarterly dummies, consider Dish's comments on the seasonality of its operations in its 2015 annual report: "*Historically, the first half of the year generally*

[14]While Dish Network was technically incorporated in 1980, the relevant starting date for our analysis is when Dish actually commenced commercial operations and could thus begin acquiring customers.

[15]In Q1 2015, 0.9% of Dish's revenue were derived from equipment sales, which are not core to the business and have not been growing over time. Dish has made investments in wireless spectrum over the past three years — wireless spectrum is a non-operating asset — but earns no revenue from it, and the core operations of the business do not depend upon it.

[16]The "Great Recession" started December 2007 and ended June 2009 (`http://www.nber.org/cycles.html`).

produces fewer gross new subscriber activations than the second half of the year, as is typical in the pay-TV industry. In addition, the first and fourth quarters generally produce a lower churn rate than the second and third quarters."

Table 2: Parameter Estimates: Dish Network

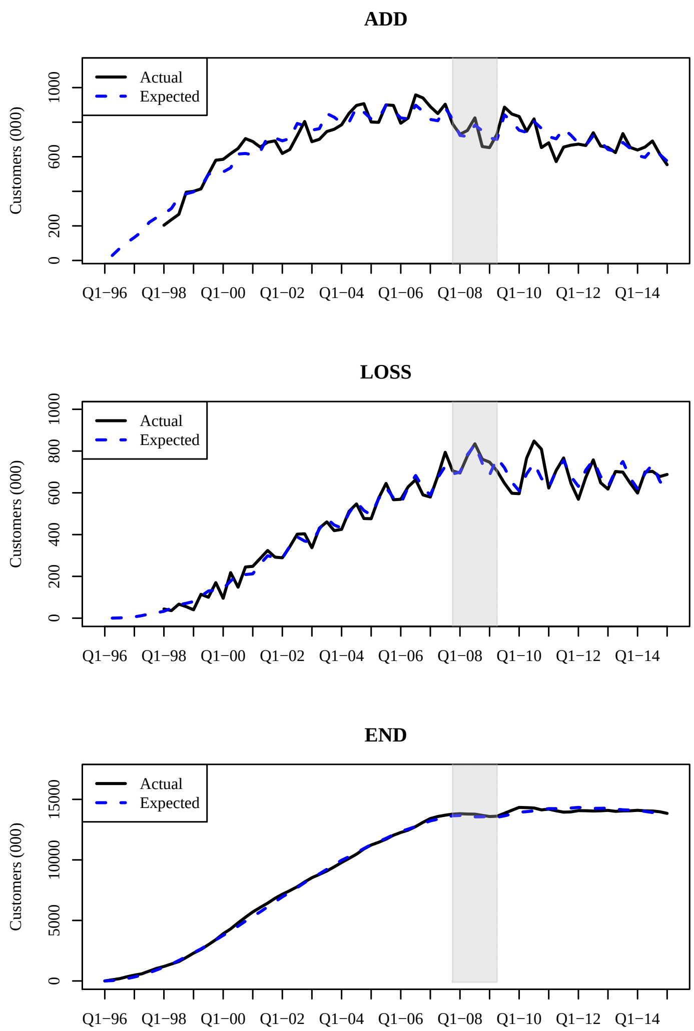
	Acquisition		Retention	
	Param.	Std. Err.	Param.	Std. Err.
r	11.440	5.123	1.648	0.232
α	29861.183	12406.027	81.196	4.897
c	2.001	.011	1.423	0.056
β_{Q1}	$-.052$	.008	$-.079$	.007
β_{Q2}	$-.057$	.007	.036	.009
β_{Q3}	.036	.008	.107	.008
β_{Rec}	$-.099$	.011	.129	.010
π_{NA}	.525	.006		

The negative effect of the 2008 recession on Dish's financials is unmistakeable; its effect upon acquisition and retention propensities was greater than all of the respective seasonal terms. The coefficient in the acquisition model is negative because customers have a lower propensity to acquire services during recession, while the coefficient in the retention model is positive because customers have a higher propensity to churn during recession.

In Figure 2, we plot model estimates for gross customer additions, losses, and end-of-period total customer counts against what we actually observed. (The grey area indicates the duration of the Great Recession.) We must backcast gross customer additions and losses because Dish did not disclose ADD and LOSS data prior to Q1 1998. Our resulting fits are good; we see a clear seasonal pattern within acquisitions and losses, and lower acquisitions and higher losses during the recession of 2008. Dish appears to be past the point of peak adoption, a sentiment echoed by Dish CEO Charlie Ergen in Dish's Q1 2015 conference call: "*My general sense is that the linear pay television business probably peaked a couple of years ago and that it's in a very slight decline.*'

Average revenue per user is modeled as per Section 4.4. We assume linear growth, which is consistent with comments made in Dish's annual financial reports. First fitting the sim-

Figure 2: Dish Network: Customer Additions, Losses and End-of-Period Customer Counts (Recession in Grey)



ple time-trend regression given in Equation 22, we find that the model residuals fail the Augmented Dickey Fuller unit root test ($t = -2.6, p = .31$). We therefore model ARPU using the ARIMA(0,1,0) model specified in Equation 23, with $\hat{\beta}_0 = 0.246$ (s.e. .091) and an associated R^2 of 93%.

5.1.2 Predictive Validation and Comparison

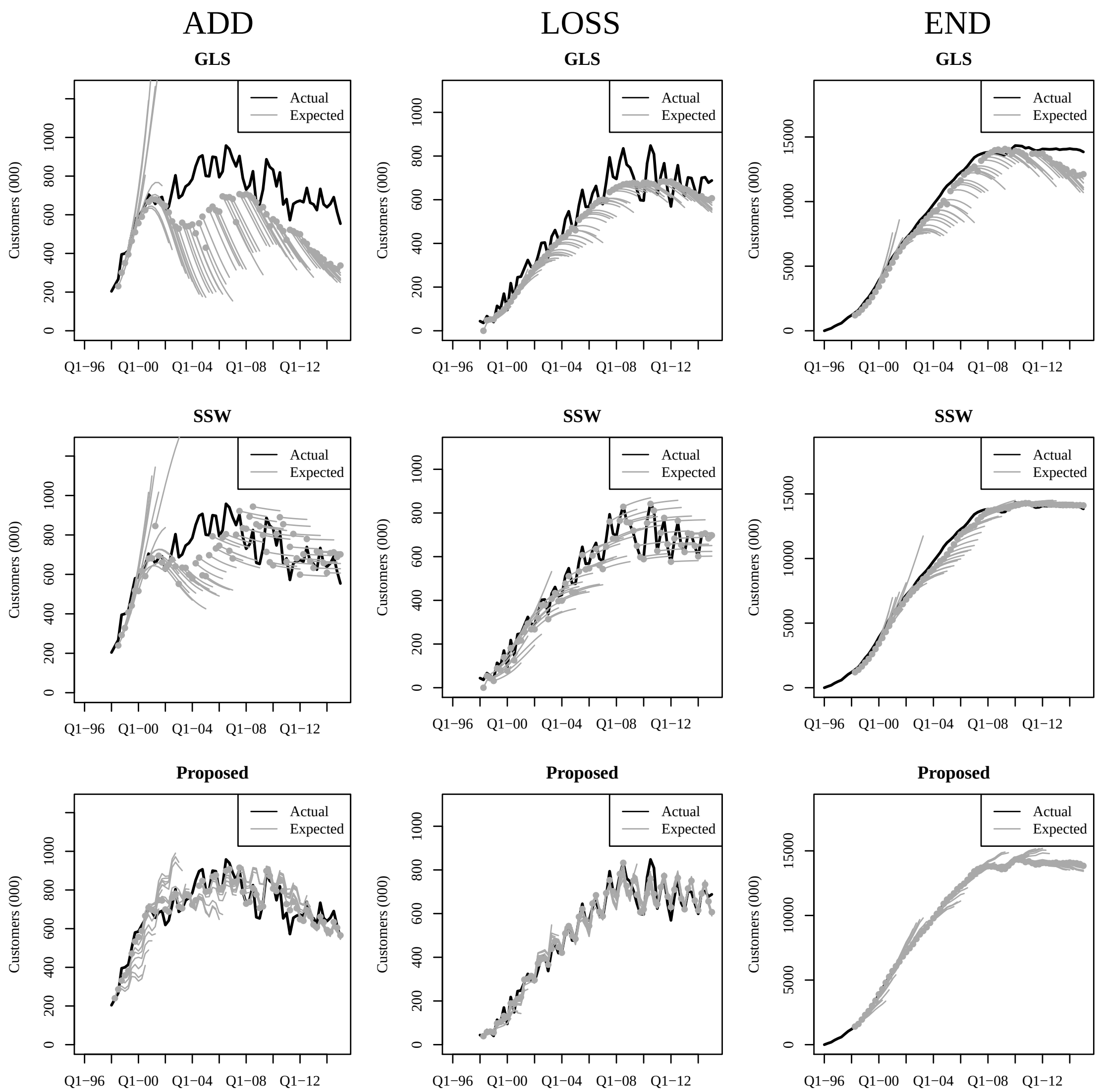
While the analysis presented above shows that our in-sample fit is very good, it does not give us any real insight into the predictive validity of our model or how our model's predictions compare to those of alternative models (i.e., those presented in GLS and SSW). These are important questions, as the quality of our estimate of firm value is a direct function of the quality of the projections of revenue (and customer acquisitions) coming from our model.

To shed light on these questions, we perform a rolling validation in which we vary the model calibration period and compare the model predictions of ADD, LOSS, and END with the actual numbers reported by Dish. Letting $Q = 10, 11, \ldots, 69$ (corresponding to all possible calibration periods ending from Q2 1998 to Q1 2013), we calibrate our model upon all data up to and including quarter Q, and then predict ADD, LOSS, and END for the next two years (i.e., ADD_{Q+q^*}, LOSS_{Q+q^*}, and END_{Q+q^*} for $q^* = 1, 2, \ldots, 8$). Because of missing data, only 3 quarters of ADD and LOSS data are available when $Q = 10$, making it a reasonable starting point to the rolling analysis. As a result, our evaluation of model performance is based on predictions made using 60 different calibration periods.

In Figure 3, we plot all resulting predictions over all calibration periods for ADD (first column), LOSS (second column), and END (third column) using GLS (first row), SSW (second row), and our proposed model (third row). While the general patterns of over- and under-estimation are similar for SSW and GLS, we see that the overall predictive validity of SSW is generally better than that of GLS. GLS underestimates future ADD, LOSS, and END figures, often severely so. This is primarily because the logistic internal-influence model for ADD and constant retention rate model for LOSS are unable to capture the underlying dynamics in customer behavior over time. Since SSW models END (rather than ADD) using the logistic internal-influence model, their resulting predictions for END are generally quite well-behaved and well-calibrated. Both methods have the most difficulty forecasting ADD, as evidenced by the large deviations between the predictions in grey and the actual data in black. This is important as ADD is an important input for these models' respective valuation models.

In contrast, our proposed model forecasts ADD, LOSS, and END very accurately, as evidenced by the tight correspondence between the grey and black lines in the bottom row of Figure 3. In contrast to the forecasts associated with the GLS and SSW models, this correspon-

Figure 3: Dish Network: Rolling Two-Year Predictive Validation Plots



dence remains tight even for short calibration periods, which is further proof of the robustness of the model's predictions.

To summarize the relative performance of these three models, we compute the absolute percentage error in the ADD, LOSS, and END forecasts for each of the (rolling) eight holdout quarters and take the average across the 60 different calibration periods. The resulting MAPE numbers are reported in Table 3. We see that the MAPE figures associated with the SSW model are generally half those of the GLS model, while the MAPE figures for our proposed method are generally one third smaller than those of SSW.

Table 3: DISH: MAPE of Predictions of ADD, LOSS, and END by Forecasting Horizon

Quantity	Horizon	GLS	SSW	Proposed
ADD	Q+1	26.0	14.7	8.2
	Q+2	29.2	16.4	9.5
	Q+3	32.5	16.4	10.4
	Q+4	36.5	16.3	11.3
	Q+5	41.1	17.7	13.2
	Q+6	45.5	19.8	14.5
	Q+7	50.6	21.5	15.6
	Q+8	55.8	22.9	16.0
LOSS	Q+1	26.0	14.7	8.2
	Q+2	29.2	16.4	9.5
	Q+3	32.5	16.4	10.4
	Q+4	36.5	16.3	11.3
	Q+5	41.1	17.7	13.2
	Q+6	45.5	19.8	14.5
	Q+7	50.6	21.5	15.6
	Q+8	55.8	22.9	16.0
END	Q+1	26.0	14.7	8.2
	Q+2	29.2	16.4	9.5
	Q+3	32.5	16.4	10.4
	Q+4	36.5	16.3	11.3
	Q+5	41.1	17.7	13.2
	Q+6	45.5	19.8	14.5
	Q+7	50.6	21.5	15.6
	Q+8	55.8	22.9	16.0

These conclusions are not affected by the fact that our model incorporates the effects of covariates while the model of GLS and SSW do not. We created variants of the GLS and SSW models that include the quarterly seasonality and Great Recession effects (through a logit formulation for retention, and a proportional hazards specification for acquisition (GLS) and ending customers (SSW)), and do not observe any significant changes to our conclusions regarding the relative performance of the three models.

5.1.3 Valuation Results

Have demonstrated the performance of our proposed model, we now to turn to the primary reason why it was developed in the first place: computing an estimate of the value of the firm.

We first project revenues (Section 4.5) far enough into the future so that all subsequent profits/losses have no effect on our valuation; we choose 50 years. We forecast that POP will continue to grow at a per-month growth rate of 0.06% into the future; this is equal to the historical monthly US household growth rate over the period from March 1996 to March 2015.

Our revenue projections drive detailed financial projections that are used to estimate future free cash flows, the weighted average cost of capital, the value of non-operating assets, and net debt. We then add the value of the operating assets to the non-operating assets and subtract the net debt to arrive at our best estimate of shareholder value using Equation 1 — see Table 4.

We estimate a stock price of $64.62 based on Q1 2015 results, which were disclosed on May 11, 2015. The end-of-day stock price that day was $66.38, implying that we are within 3% of the then-current stock price. Holding all else constant, the Dish Network stock price estimates computed using the GLS and SSW models for customer acquisition and retention were $48.84 and $63.72, respectively.

Table 4: Dish Valuation Summary (End of Q1 2015)

Value of Operating Assets	$15.7B
Non-Operating Assets - Net Debt	$14.1B
Shareholder Value	$29.9B
Shares Outstanding	462.1MM
Implied Stock Price	$64.62
Actual Stock Price	$66.38
Over(under)-estimation	(2.7%)

Our valuation and the corresponding implied stock price are point estimates. So as to get a sense of the uncertainty in these estimates, we undertake the following sensitivity analysis. First, holding the parameters of the retention and ARPU processes constant, we draw a new set of parameter values for the acquisition process model via bootstrap resampling of the model residuals (Efron and Tibshirani 1993, Chapter 9). Given this set of parameters, we compute the resulting revenue numbers, the corresponding estimate of the value of the firm and the implied stock price. We do this for 500 draws and compute a 95% interval for our implied stock price. We repeat this for the retention process (holding the parameters of the acquisition and ARPU processes constant) and the ARPU process (holding the parameters of the acquisition and retention processes constant). The interval associated with the acquisition process is [$64.48, $64.77] ($+/-0.2\%$). The equivalent intervals for the retention and ARPU processes are [$62.47, $66.78] ($+/-3.4\%$) and [$62.76, $66.49] ($+/-3.0\%$), respectively. This suggests that it would be most beneficial to investors if Dish were to provide more or better data regarding customer retention (e.g., by disclosing LOSS figures monthly instead of quarterly).

5.2 Sirius XM

To test the robustness of our framework, we repeat our valuation exercise for a second company, Sirius XM, which is a broadcasting company that provides satellite radio services in the United States. Sirius XM is a good complementary example to that of Dish for a number of reasons:

1. Sirius XM is a relatively high-growth business, while Dish is a mature business. We note that ADD, LOSS, and END are all past their peak for Dish (Figure 2), while they are increasing for Sirius XM.

2. Sirius XM suffers from more severe missingness than Dish. Sirius XM was formed by the merger of Sirius Satellite and XM Satellite, which began commercial operations in February 2002 and November 2001, respectively. Neither Sirius nor XM disclosed ADD, LOSS, or END data for paying customers. It was not until after the merger that these data were first publicly disclosed (Q3 2008). As a result, almost half of Sirius XM's customer data is missing.

3. Sirius XM is a high fixed-cost business because its satellite radios are pre-installed in most new vehicles, while Dish Network is a high variable-cost business. Most of Sirius XM's operating expenses, net of subscriber acquisition costs (SAC), are fixed in nature, while most of Dish's operating expenses are variable. All else being equal, this substantially increases the marginal profitability of new Sirius XM users.

4. Sirius XM has a very different customer base and customer profile than Dish. Sirius XM has a larger number of customers, each of whom generates less revenue but is much cheaper to acquire.

5. Sirius XM sells almost entirely into cars, whereas Dish sells almost entirely into homes. All else being equal, this makes Sirius XM a more cyclical business than Dish.

See Table 5 for a comparison of the two companies on the basis of some basic measures.

Table 5: Comparison of Dish and Sirus XM (at point of valuation)

	Dish	Sirius XM
Total Paying Customers	13.8MM	22.9MM
Monthly ARPU	\$88.72	\$16.72
Weighted Average Cost of Capital	7.2%	6.9%
SAC / Customer	\$716.46	\$82.06
ARPU Growth Per Year	\$2.95	\$0.49

Despite these differences, we proceed with virtually the same model. The main change is that the population unit for Sirius XM is cars (as opposed to households for Dish). The market size for Sirius XM is equal to the number of vehicles on the road, as provided by the Bureau of Transportation Statistics. Correspondingly, we use vehicle sales, as defined/provided by Federal Reserve Bank of St. Louis, as our macroeconomic covariate. We denote the coefficient associated with the vehicle sales covariate by β_{VS}.

The parameter estimates of the acquisition and retention process models are reported in Table 6; the associated model SSE is $146{,}799$.[17] Once again, we assume linear growth when modeling average revenue per user. Fitting a simple time-trend regression (Equation 22) to the data, we find that the residuals do not fail the Augmented Dickey Fuller unit root test (test statistic: $t = -3.56, p = .04$); the associated parameter estimates are $\hat{\beta}_0 = 9.643$ (s.e. .218) and $\hat{\beta}_1 = 0.041$ (s.e. .002), with $R^2 = 88\%$.

In Figure 4, we plot model estimates for ADD, LOSS, and END against what we actually

[17]Unlike Dish, the Weibull-Gamma baseline retention process for Sirius XM is not significantly different from a Weibull baseline. We retain the more general formulation for consistency with Dish.

Table 6: Parameter Estimates: Sirius XM

	Acquisition		Retention	
	Param.	Std. Err.	Param.	Std. Err.
r	0.208	.239	153.206	35.129
α	7983.761	9213.648	2671.833	602.632
c	2.228	.154	1.066	.085
β_{Q1}	$-.106$	.012	.029	.014
β_{Q2}	$-.049$	.014	$-.039$	.016
β_{Q3}	$-.047$	.013	.003	.015
β_{VS}	.077	.003	$-.013$	.003
π_{NA}	.011	.075		

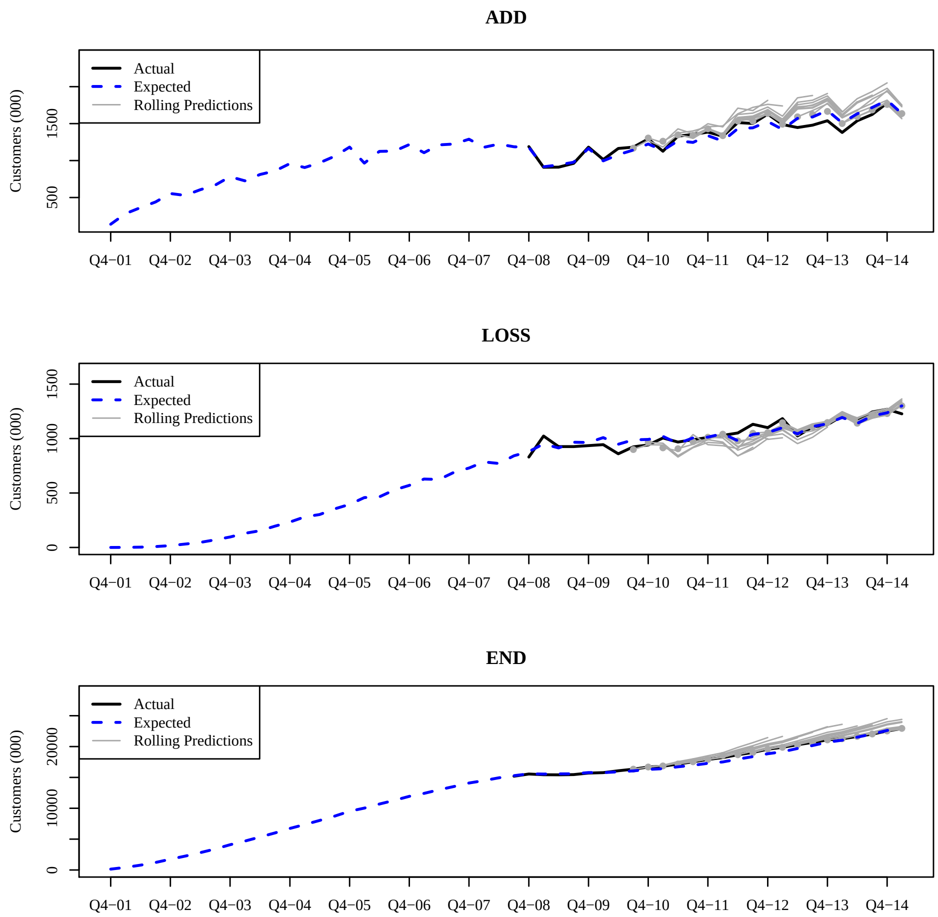
observed. We overlay a set of two-year rolling predictions corresponding to all possible calibration periods ending from Q3 2010 to Q1 2013, as we had done for Dish in Section 5.1.2. As was the case with Dish, the in-sample and out-of-sample fits for Sirius XM in terms of all three customer metrics are good.

As with Dish, we project revenues 50 years into the future. We project both future vehicles on the road and vehicle sales assuming monthly growth rates are equal to their respective historical cumulative average growth rates from 1980 until 2015. We perform a detailed margin and cash flow analysis to turn the revenue projections into monthly free cash flow projections. The resulting valuation is presented in Table 7. We estimate Sirius XM's operating assets to be worth \$27.1B. After adding non-operating assets (Sirius XM has approximately \$1.1B in net operating loss carry-forwards) and subtracting net debt, we estimate shareholder value to be \$23.4B using Equation 1. This implies a stock price of \$4.24 based on Sirius XM's Q1 2015 results, which were released on April 28, 2015. The end-of-day stock price that day was \$3.90. Holding all else constant, the Sirius XM stock price estimates computed using GLS's and SSW's models for customer acquisition and retention were \$0.41 and \$6.55, respectively.

5.3 Additional Insights

Confident that our model provides sensible valuation estimates, we return to Dish to study other insights that we are able to draw from the model beyond stock price estimates. We look at

Figure 4: Sirius XM: Customer Additions, Losses and End-of-Period Customer Counts



the remaining/residual lifetime and lifetime value of Dish customers as a function of the length of their relationship (i.e., tenure) with the firm. We then decompose Dish's current customer equity (CCE) by tenure. While these seem like fairly ordinary applications of a customer-level model, it is important to keep in mind that we are doing these analyses with no customer-level data; all we have are the aggregated summaries that companies disclose to the public.

5.3.1 Comparison of Residual Value by Tenure

Let us consider a Dish customer acquired at the end of Q1 2015 whom we call Recent Robin, and another Dish customer acquired 10 years earlier at the end of Q1 2005 whom we call Long-time Larry. One quantity of managerial interest is the expected remaining (or residual) lifetime

Table 7: Sirius XM Valuation Summary (End of Q1 2015)

Value of Operating Assets	$\$27.1$B
Non-Operating Assets - Net Debt	$-\$3.7$B
Shareholder Value	$\$23.4$B
Shares Outstanding	5513.7MM
Implied Stock Price	$\$4.24$
Actual Stock Price	$\$3.90$
Over(under)-estimation	7.4%

of Recent Robin and how it compares to the expected residual lifetime of Longtime Larry. The GLS and SSW models both assume that all customers are equal and thus Recent Robin and Longtime Larry would be expected to share the same expected future lifetime. However, we intuitively expect that Longtime Larry is likely to remain a customer for a longer period of time because his long history with Dish thus far suggests that he has a lower churn propensity. By definition, the expected residual lifetime of a customer acquired in month m who is still a customer in month M equals

$$\sum_{i=0}^{\infty} \frac{S(M+i-m|m, \mathbf{X}_R(m, M+i); r_R, \alpha_R, c_R, \boldsymbol{\beta}_R)}{S(M-m|m, \mathbf{X}_R(m, M); r_R, \alpha_R, c_R, \boldsymbol{\beta}_R)} . \tag{26}$$

(See the Appendix for details of how we perform this calculation.) The expected residual lifetime of Recent Robin and Longtime Larry are 5.5 years and 9.4 years, respectively. This difference is in line with our intuition.

Investors should be interested in the expected lifetime of customers. Longer expected customer lifetimes imply more stable future cash flows, all else being equal, because future cash flows are less reliant on the acquisition of new customers. At Dish, we see that not only do older customers have longer residual lifetimes, but also that all customers live for a relatively long time, which should be heartening to investors. Reducing investors' perceived risk of future cash flows reduces the cost of capital, raising firm valuation.

Another quantity of interest is the residual lifetime value (RLV) of customers.[18] Calculating

[18]We make the distinction between CLV, which we reserve for as-yet-to-be acquired customers, and RLV, which

this using nothing but the information provided in a firm's financial statements requires careful consideration of what expenses are fixed versus variable, and a proper handling of subscriber acquisition costs. (See the Appendix for details of how we perform this calculation.) We estimate that the (pre-tax) RLV associated with Recent Robin to be $1,426, excluding average initial acquisition costs of $854, while Longtime Larry is worth $1,932. While it is not possible to provide predictive validation of these customer insights because of the aggregated nature of the data, the predictive valuation analysis that we performed provides general validity for these results.

This information is useful to many stakeholders:

- Investors may track CLV relative to SAC per customer, viewing these metrics as financial barometers of customer health. Unfavorable trends in these figures (as has been evident at Dish, for example) could be indicative of decreasing customer (and thus firm) profitability.

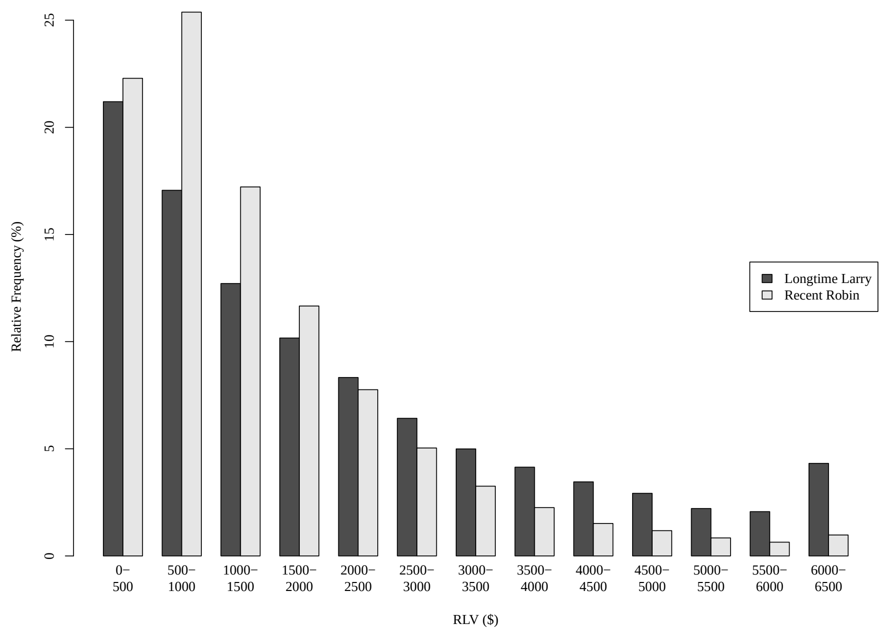
- Competitors, comparable companies, and investors will be interested in the absolute level of CLV and RLV for benchmarking purposes. If a competitor estimates its own CLV to be less than Dish's, there may be opportunities to "close the gap," identifying what it could be that is causing the gap in average customer profitability. Investors may ask the same question and demand that changes be made to improve CLV and RLV.

While the preceding analysis has focused on *expected* residual lifetime and lifetime value, we can examine the distribution of these quantities across all possible Recent Robins and Longtime Larrys. In Figure 5 we provide a histogram representing 1MM samples from their respective RLV distributions. This provides us with additional color regarding the riskiness of future cash flows associated with new and existing customers. For example, we estimate that there is a 41% chance that the company will incur a loss on a Recent Robin (i.e., 41% of Recent Robin's

applies to existing customers. Subject to minor accounting issues, these two quantities are equal when the we have constant retention rates (i.e., there is no heterogeneity across customers and/or duration dependence within customer). However, this is rarely the case and it is therefore important to make this distinction (Fader and Hardie 2010).

RLV samples (Figure 5) lie to the left of \$854, the SAC per customer for Dish). We infer a long right tail to Longtime Larry's pre-tax RLV — this drives up Longtime Larry's expected pre-tax RLV, but also implies a much higher variance about that expectation. Longtime Larry is more valuable but is also more risky (McCarthy, Fader, and Hardie 2016).

Figure 5: Histogram of 1MM Sampled RLV's — Recent Robin and Longtime Larry



5.3.2 Customer-Base Decomposition

The raw data available from virtually any public source reveals nothing about the tenure of existing customers or how these "lifetimes" vary across the customer base. As suggested by the examples of Recent Robin and Longtime Larry, this can be important information to outside investors. Fortunately, as just demonstrated, our proposed model makes it easy for analysts to infer these lifetimes. We can go further and segment the customer base on this basis.

The proportion of currently active customers (i.e., active at the end quarter Q) who were born in month m is equal to

$$\frac{\widehat{C}(m, 3Q)}{\sum_{i=1}^{3Q} \widehat{C}(i, 3Q)}. \tag{27}$$

While knowing the count of customers within each segment is helpful, the value of those customers is probably of greater interest to investors and managers. Recall that the sum of RLV across all the firm's current customers is called current customer equity (CCE). It follows that the proportion of total CCE, as at the end of quarter Q, coming from customers who were born in month m is the RLV-weighted analog of Equation 27:

$$\frac{\widehat{C}(m, 3Q)E(\text{RLV}_{m,3Q})}{\sum_{i=1}^{3Q} \widehat{C}(i, 3Q)E(\text{RLV}_{i,3Q})}, \tag{28}$$

where $\text{RLV}_{m,3Q}$ is the residual lifetime value of a customer acquired in month m who is still active in month $3Q$. The resulting decomposition of Dish's customer base is presented in Table 8. We estimate, for example, that approximately one-eighth of Dish's customer base is comprised of highly loyal/inertial customers who have been Dish subscribers for 10+ years. We also infer that longer-lived segments comprise proportionally more of the total value of the customer base because they are inferred to have higher residual lifetime values, as is evident from our comparison of Recent Robin and Longtime Larry above.

Table 8: Decomposition of Current Customer Equity (End of Q1 2015) by Tenure

Tenure (years)	% Customer base	% CCE
0–2	31	28
2–5	31	29
5–10	25	27
10+	12	16

This decomposition, and other granular inferences that can be drawn from our model, can provide useful insights for investors. In some sense, the overall corporate valuation shown earlier isn't necessarily very insightful by itself; it merely captures the "voice" of the financial market. It could be argued that the real value of our proposed approach is the ability to go beyond the macro valuation estimate to offer useful, operational diagnostics to better understand where that value is coming from, and what it might mean to the firm, its competitors, suppliers, investors, and possibly even public policy makers. In the case of Dish, the considerable amount of value coming from long-lived customers is indicative of a very mature business, and implies that the valuation of the business as a whole will be much more dependent on and sensitive to changes in the company's ability to retain existing companies, rather than acquire new ones. In contrast, Sirius XM's valuation is far more dependent on the the firm's ability to acquire new users and to earn a high rate of return on the firm's investment in those new users. This is important information for investors and managers alike.

6 General Discussion and Future Work

As noted at the outset of the paper, our objective has been to create a realistic model for customer acquisition and retention (which can be estimated using data publicly reported by firms with a subscription-based business model), and embed it within a standard financial framework for corporate valuation. Looking beyond the methods developed here, we hope this paper will serve as a call to action for firms, analysts, and investors to perform these kinds of analyses on a more regular and rigorous basis. We have provided several use cases for the insights that can be derived from our analyses, including but not limited to comparing CLV across comparable/competing firms, performing customer value segmentation, and providing investors with improved forward-looking sales visibility. All of this is possible because we have performed our valuation using a flexible, general-purpose model of customer behavior in contractual/subscription settings.

While our model is particularly suited to third parties analyzing publicly traded companies using their public disclosures, we contend that this same exercise can — and should be — undertaken internally as well. Firms can easily implement an equivalent version of this model using internal company data, enhancing its overall validity. While the estimation procedures differ slightly (given access to more granular data), the models for customer acquisition, retention, and spending, along with the proposed valuation framework, would remain essentially the same. Measuring and tracking CLV and RLV can improve the ROI of a company's acquisition and retention spending, and our valuation framework gives company executives the ability to estimate how much value this ROI improvement has created for the overall value of the firm. This provides executives with an important key performance indicator to hold themselves and their marketing managers accountable to.

While our model is more flexible than previously published customer-based corporate valuation models (e.g., in terms of the dynamics that it can accommodate), it has nevertheless remained parametrically parsimonious because the available data is limited — and will likely stay that way for the foreseeable future. For example, it is highly unlikely that firms will begin to disclose the kinds of data required to properly account for other sources of customer value, such as the referral value of a customer (Kemper 2010; Kumar et al. 2010; Kumar, Petersen, and Leone 2007), the impact of social media (Luo, Zhang, and Duan 2013; Yu, Duan, and Cao 2013), customer satisfaction (Anderson, Fornell, and Mazvancheryl 2004; Homburg, Koschate, and Hoyer 2005; Luo and Bhattacharya 2006), or heterogeneity in the spend per customer (McCarthy, Fader, and Hardie 2016). At the same time, indirect proxies for these factors may be obtainable in some cases through external data sources for a small subset of companies.

Furthermore, it may seem tempting to add in other "bells and whistles" to further enrich the model specification used here. We should be open to such possibilities but are cautious about our ability to do so. For instance, it may be the case that individual-level acquisition and retention propensities are correlated (i.e., customers who take longer to acquire may have a lower propensity to churn once they have been acquired or vice versa — see Schweidel, Fader, and Bradlow (2008a) — but our ability to empirically identify such a correlation is very limited, increasing the risk that we over-burden the limited data we have available. Many other theoretically reasonable extensions (e.g., allowing for cross-cohort effects (Gopalakrishnan, Bradlow, and Fader 2016), or specifying a more complicated market potential model) will likely suffer from similar issues. Bodapati and Gupta (2004) warn us that when data is highly aggregated, even identifying heterogeneity (in their setting, using a finite mixture model) can be challenging. Model parsimony is a good thing.

Another limitation we readily acknowledge is that our framework is appropriate only for subscription-based businesses, for whom attrition is *observed* (Fader and Hardie 2009). For noncontractual settings (where attrition is unobserved), we can specify an alternative model for the number of customers matrix. However, it is not clear what kinds of metrics a company operating in such a setting would have to disclose in order for an outside analyst to be able to estimate $C(\cdot, \cdot)$ and perform the valuation task in a valid manner. This is an important area of future research.

We have focused our attention strictly upon conducting the valuation process for one company at a time, but our predictive accuracy may be improved if we were to develop a hierarchical Bayesian model, estimating the parameters for many companies at the same time. This may alleviate some of our data inadequacy issues by "borrowing strength" across firms, but will require considerable methodological advancements to properly "share" information across firms and handle aggregated missing data.

Beyond the methodological improvements, our valuation framework could provide perspective to the ongoing discussion among marketing scholars regarding the accounting of customer equity and advertising spending. Consistent with Srinivasan (2015), the vast majority of Dish's SAC is expensed and not capitalized (82% in Q1 2015) — the primary component of SAC that is capitalized is spending to purchase satellite receivers, which are then owned by Dish and depreciated over a useful life of approximately 4 years. In contrast, just acquired customers have, on average, a longer "useful life" of 5.5 years (Section 5.3), and yet are not considered assets (Wiesel, Skiera, and Villanueva 2008). As a result, subscriber acquisition activities create costs that are incurred immediately but whose benefits are received in the future; as such, the income statement is not reflective of the underlying economic condition of the business. It is no surprise, then, that Dish was generally unprofitable earlier in its history, and profitable in recent years.

As companies increasingly recognize the importance and merit of customer-centric business strategies (Fader 2012), and in turn disclose customer data on a more regular and thorough basis, there will be a growing opportunity for marketing scholars to study the behavior of large, publicly traded companies through their customer data in conjunction with their financial statements. We hope that this paper lays a sound foundation for how future analyses will incorporate, and shed further light on, company valuation.

Appendix

RLV can be expressed mathematically as

$$E(RLV) = \int_{t'}^{\infty} E[V(t)]S(t|t > t')d(t - t')\, dt\,,$$

where t' is the age of the customer at the point in time where their residual lifetime value is computed, $E[V(t)]$ is the expected net cash flow of the customer at time t (assuming they are alive at that time), $S(t|t > t'))$ is the probability that the customer has remained alive to at least time t (given they were alive at t', and $d(t-t')$ is a discount factor that reflects the present value of money received at time t (Fader and Hardie 2015).

This is acceptable as a mathematical representation of the definition of (expected) RLV but is of limited use in practice, as it ignores the accounting issues identified in Section 5.3.1. However, these accounting issues are considered when performing our valuation and an intermediate result from these calculations can be used to compute RLV. The intermediate result of interest is EBITDASAC, earnings before interest, taxes, depreciation, amortization, and subscriber acquisition costs.

Let $\tau_{m,M}^{(k)}$ be the kth sampled residual lifetime of a customer acquired in month m who is still active in month M, which is drawn from the distribution

$$P(T_{m,M} = \tau) = \frac{S_R(\tau + M - m - 1|m) - S_R(\tau + M - m|m)}{S_R(M - m|m)}, \quad \tau = 1, 2, \ldots . \tag{A1}$$

The expected residual lifetime of a customer acquired in month m who is still active in month M can be computed as the average of many samples drawn from this distribution:

$$E(T_{m,M}) \quad = \quad \frac{1}{K} \sum_{k=1}^{K} \tau_{m,M}^{(k)}\,. \tag{A2}$$

We set $K = 1{,}000{,}000$ in our analysis.

Given monthly EBITDASAC numbers, the value of an average customer in month $M + m^*$

is

$$\frac{\text{EBITDASAC}(M+m^*)}{\left[\widehat{C}(.,M+m^*-1)+\widehat{C}(.,M+m^*)\right]/2}\,.$$

Therefore, the pre-tax RLV of a customer with sampled residual lifetime $\tau^{(k)}_{m,M}$, is

$$\text{rlv}^{(k)}_{m,M} \;=\; \sum_{m^*=1}^{\infty} \frac{\text{EBITDASAC}(M+m^*)}{\left[\widehat{C}(.,M+m^*-1)+\widehat{C}(.,M+m^*)\right]/2}\frac{\mathbb{1}(m^*\leq \tau^{(k)}_{m,M})}{(1+\text{WACC})^{m^*}}, \tag{A3}$$

(The empirical distribution of these draws is plotted in Figure 5.) Averaging over these sampled realizations of residual lifetime gives us the expected RLV of a customer acquired in month m who is still active in month M,

$$E(\text{RLV}_{m,M}) \;=\; \frac{1}{K}\sum_{k=1}^{K}\text{rlv}^{(k)}_{m,M}\,. \tag{A4}$$

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