Market Power in the Securities Lending Market*

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Abstract

We document the presence of market power in the equity securities lending market and evaluate its impact on different investor groups and valuations. Our analysis reveals high market concentration, non-competitive fees, and low inventory utilization in the cross-section of stocks. Motivated by this evidence, we develop and estimate a dynamic asymmetric-information model that sheds light on the benefits of this current market structure for both security lenders and short sellers. We find that lending fee income raises shares lenders' equity valuations by 1.5% for large-cap, low-fee stocks, by up to 25% for small-cap stocks, and by even more than 100% for nano-cap stocks. Our model yields estimates of the distribution of alphas from shorting different segments of the cross-section of stocks, indicating that fees reduce short sellers' profits by about 60%.

Keywords: Short selling, market power, custodian lenders

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1 Introduction

Short selling in financial markets regularly attracts the attention of investors and regulators, especially during times of extreme stock price movements. In this paper, we investigate how the market structure of the securities lending market affects different investor groups and their valuations. To start, we empirically document that the equity securities lending market in the U.S. is highly concentrated. This concentration is the result of the delegation of securities lending to a small set of intermediary custodian lenders. For a given stock, the top two security lenders typically command a large market share, ranging between 40% and 70%.[1] Further, consistent with the presence of market power, we document that fees on lending contracts are elevated and non-competitive essentially across the whole universe of publicly traded stocks. Moreover, we show that a lack of sufficient inventory is generally not the culprit for high fees, as fees exceed lenders' marginal cost irrespective of inventory utilization.

Motivated by this evidence, we develop and estimate a dynamic model of trading that fulfills two main objectives: first, it conceptually clarifies the impact of market power on lending fees and sheds light on the prevalence of a delegated market structure for securities lending in practice. Second, the model yields quantitative estimates of the impact of non-competitive fees on security lenders' stock valuations and of the distribution of alphas that short sellers obtain from targeting different segments of the cross-section of stocks.

Our results reveal a substantial impact of fee income on security lenders' stock valuations, ranging from 1.5% of extra value for large-cap, low-fee stocks to value inflations up to 25% for small-cap stocks, and even more than 100% for nano-cap stocks.[2] These excess valuations do not reflect stock fundamentals in the traditional sense, that is, they are

[1] See Section 2 for details.

[2] See Cherkes et al. (2013) for a related analysis specifically highlighting the role of fee income in explaining the Palm-3Com spin-off puzzle.

distinct from the present value of a stock's future dividends. Instead, these value wedges represent a transfer of rents from informed short sellers to shares lenders via fee payments. As such, our results reveal that market power in securities lending markets can have material spill-over effects on valuations in stock markets, distorting the informational content of prices.

Our estimated model also provides estimates of the hard-to-observe distribution of alphas that short sellers (e.g., hedge funds) obtain from targeting different segments of the cross-section of stocks. We find substantial cross-sectional variation in net-of-loan-fee alphas across size and fee groups. Our findings indicate that fees reduce short sellers' profits typically by about 60%. Finally, the market-power channel present in our framework contributes to low inventory utilization as observed in the data and an inelastic response of the equilibrium quantity of lending with respect to before-fee shorting profits.

We present a dynamic model of trade under asymmetric information with two trading venues, a centralized limit order market akin to Glosten and Milgrom (1985) and a securities lending market facilitating short selling. For the latter market, we compare two market structures: a delegated and opaque over-the-counter market that resembles the status quo and a centralized market that is transparent and competitive.

Key elements of our model are informed traders' concerns about information leakages and a recognition of the fact that shorting is a two-step transaction whereby securities can be sold only after they have been borrowed.[3] As a result, short sellers are concerned about the transparency of the securities lending market, especially when the underlying security is illiquid. If securities lending activity is organized in a centralized market and thus publicly observable, the resultant information leakage impedes traders' ability to profit from shorting: stock prices are immediately updated to incorporate those signals from the se-

[3]Regulations in practice ban naked short selling, except for short sales by market makers engaged in bona fide market making activities (see SEC, 2022).

curities lending market, leading to price adjustments before traders can actually sell the securities they just borrowed. Thus, short sellers benefit from the intransparent nature of a delegated securities lending market when they establish their positions. Moreover, this lack of transparency is in the interest of shares lenders, who can extract a fraction of short sellers' information rents via lending fees. However, not all market participants are better off under this market structure. Liquidity traders are negatively affected, as they are the source of surplus flowing to hedge funds and securities lenders.

Using this dynamic framework, we quantitatively evaluate the implications of non-competitive fees for security lenders' stock valuations and the distribution of alphas short sellers obtain from targeting different segments of the cross-section of stocks. We show that to estimate these effects, it is of first-order importance to properly capture the joint dynamics of stocks' dividend and fee income.[4] Our estimated model accounts for these dynamics based on a granular state specification that captures firms' transitions in the cross-sectional size and fee distributions. The resultant estimates reveal that lending fee income has first-order effects on security lenders' stock valuations, especially for smaller stocks.

Related literature. Our paper is related to work striving to understand implications of short selling in financial markets. A significant part of the theoretical literature in this regard has focused on implications of short-sales constraints on prices; examples include Miller (1977), Harrison and Kreps (1978), Diamond and Verrecchia (1987), Hong and Stein (2003), and Scheinkman and Xiong (2003). In contrast, our work is part of a fairly small subset of papers that considers the determinants and consequences of shorting fees.

In modeling lending markets and investigating the impact of their presence on security prices, Blocher et al. (2013) take a reduced-form approach, making assumptions on properties of demand functions in the equity and lending markets. Duffie et al. (2002) and

[4] Engelberg et al. (2018) also highlight the importance of the empirical fact that fees are not static but rather evolve stochastically over time.

Vayanos and Weill (2008) rationalize the emergence of lending fees utilizing search and bargaining-based models with heterogenous beliefs. In these models, the degree of heterogeneity of beliefs plays a role in explaining the magnitude of lending fees, with higher heterogeneity entailing higher shorting fees and subsequently higher prices and lower future expected returns. Garleanu et al. (2021) provide a model with heterogenous beliefs and matching costs in the shorting market that explains how fears among short sellers can become self-fulfilling and lead to run-type behavior.

Atmaz et al. (2019) consider a differences-in-opinion model where only a subset of optimistic traders can take the other side of pessimists' short positions. In that case, an additional payment to these overexposed agents (a fee) is necessary to ensure that they agree to the same stock price as other optimists with the same beliefs that are not overexposed (who cannot lend and thus are not receiving fee income). In this competitive setting, positive lending fees go hand-in-hand with a fully utilized lendable inventory. Evgeniou et al. (2019) also consider a heterogeneous beliefs model, but assume that lending occurs through a monopolistic custodian lender. In their setting, every share that is available for lending is also shorted.

In Banerjee and Graveline (2014) short selling needs emerge due to endowments with opposite exposures to a fundamental shock. Due to regulatory restrictions and market frictions, constraints on borrowing and lending can be binding. They show this implies derivatives are no longer redundant and can reduce associated price distortions.[5]

Our paper contributes to this literature on the conceptual and on the empirical and quantitative side. Conceptually, we highlight two key results: (1) We show that information asymmetries between traders and corresponding concerns about information leakages imply that even hedge funds may benefit from a delegated and opaque securities lending

[5]Other papers that model the stock lending market include Duffie (1996), the working paper version of D'Avolio (2002), Nutz and Scheinkman (2020), and Nezafat and Schroder (2021).

market that is subject to market power. (2) The strategic fee choice in our setting is consistent with the empirically observed elevated fee levels in conjunction with slack lender inventory. Quantitatively, our model yields novel estimates of the impact of non-competitive fees on security lenders' stock valuations and the distribution of alphas that short sellers obtain from targeting different segments of the cross-section of stocks.

In the following, we discuss in more detail the elements of our model that differentiate it from the above-discussed literature. The notion that short sellers are concerned about information leakages is generically absent in differences-in-opinion settings, as agents agree to disagree in these models. Thus, a centralized and transparent market structure would generally be best suited to facilitate trade and would eliminate positive fees. To introduce positive fees, these settings then typically introduce one of two assumptions: (1) the presence of technological matching frictions that apply specifically to shorting activity, or (2) a lending supply that is scarce and fully exhausted by borrowing demand.

Regarding assumption (1), it is useful to note that existing trading and settlement technologies in many financial markets render it technologically feasible to match demand and supply at high speed and very low cost (as is done for other types of transactions, such as regular equity trades). Thus, it is difficult to rationalize double-digit fee levels observed in practice with exogenously assumed technology frictions. We highlight that the opaque OTC structure may rather be deliberately chosen by market participants; it is not due to a lack of an availability of extremely efficient matching and settlement technologies.[6] Regarding assumption (2), our empirical evidence regarding low inventory utilization is inconsistent with the notion that positive lending fees primarily emerge due to a scarce lendable supply. Rather, the evidence points to strategic rationing due to the presence of market power, as discussed in the next section.

[6] The securities lending market is extremely opaque even ex-post. For example, for most investors, it is infeasible to even obtain information on the market share of lending revenues of individual custodian lenders.

2 Empirical Evidence of Market Power

In this section, we present evidence of market power in the U.S. securities lending market based on several key indicators. Our securities lending data are from Markit Securities Finance Data Analytics (see Appendix C for a detailed data description). Throughout this paper, we are interested in both aggregate measures and cross-sectional heterogeneity. To this end, we present most of our results by categorizing the cross-section of firms into 25 groups. We first sort firms by size (equity market capitalization) and then into subgroups that vary by fees. The size categories are created based on widely used Russell indices and are labeled large-cap (top 1-200), mid-cap (201-1000), small-cap (1001-2000), micro-cap (2001-4000), and nano-cap (all remaining stocks).[7] For each of those size groups, we further sort stocks according to their fee yield, where the fee yield is conceptually the equivalent of a dividend yield but capturing cash flows from securities lending rather than from dividends. Specifically, it is defined as the ratio of a stock's total annualized fee income to the market value of the stock's posted inventory. As we will show in our analysis below, the fee yield is a central measure entering lenders' stock valuations. Using this measure, we assign stocks in each size bin to five fee-yield bins with the percentile cutoffs 80%, 90%, 95%, and 98%. We provide a detailed description of this approach in Appendix C.1.

Market concentration. Tables 1 and 2 document the market shares of the top two security lenders for each of our 25 firm groups. Table 1 documents the within-group average of the market shares that are reported by Markit, weighting stock-specific market shares by each stock's relative contribution to the total value on loan in a given group. These estimates indicate that the top two lenders tend to have a large market share, ranging between

[7] An advantage of using the Russell indices is that they are widely-used US equity benchmarks for institutional investors that are major lenders and borrowers in the U.S. securities lending markets.

50% and 85%.

As a robustness check, we adjust our market share measures in Table 2 in two ways. First, within each group, we weight the market shares of the top lenders for a given stock by the stock's share of the total fee income generated by stocks in the group (rather than using the value on loan). This approach thus puts more weight on the market shares of stocks that are important contributors of fee income. Second, we adjust the Markit-reported market share numbers to account for the fact that Markit does not cover all securities lending activity. Markit reports that it covers approximately 85% of lending activity. We therefore scale down the Markit-reported market shares by this number. This approach assumes that any lending activity not accounted for in the market share numbers reported by Markit involves institutions other than the top two lenders identified by Markit. This more conservative approach can be viewed as providing lower-bound estimates. Yet, we still find that the top 2 lenders typically command a combined market share between 40% and 70% of the stocks on loan.

Pricing of loan contracts. Apart from market concentration, we investigate evidence from equilibrium pricing in the securities lending market. Table 3 shows that fees on loan contracts range between 0.29% per annum for large-cap, low-fee stocks to 75.14% for nano-cap, high-fee stocks. Even small-cap (mid-cap) stocks have fees ranging between 0.29% and 12.24% (0.28% and 6.88%) per annum. When interpreting these prices for lending contracts, it is important to recognize that lenders collect these fees in addition to dividend payments. As shown in our theoretical analysis below, positive fees are inconsistent with a competitive benchmark, provided that there is excess inventory for a given stock. After all, if the supply of lendable inventory exceeds the demand, competitive lenders cannot demand incremental compensation for lending securities, as they do not sacrifice any source of cash flows (dividends) and do not take additional risk.[8]

Going beyond cash flow rights, one might be concerned that elevated fees represent compensation for voting rights, which a shares lender passes on to the borrower. To examine the impact of this channel, we report in Panel A of Table 14 (see Appendix E) fees just like in Table 3 but conditional on removing all observations 15 days before and after voting record dates to eleminate the potential effect of voting rights on lending fees. Comparing Tables 3 and 14 reveals that voting rights have hardly any impact on the magnitudes of loan fees. For example, whereas the highest-fee micro-cap stocks have a typical fee equal to 38.43% in our baseline specification, this number is 37.58% once we exclude days around voting record dates.[9]

Finally, as another robustness analysis, we examine the potential role of shares transfers around ex dividend dates, for example due to concerns about a differential taxation of investors. In Panel B of Table 14 we report fees conditional on removing observations 15 days before and after ex-dividend dates. Just like in the case of voting record dates, the results for fees throughout the whole cross-section are hardly affected. Going back to the example of the highest-fee micro-cap stocks, we find that the typical fee is now equal to 39.18% as compared to 38.43% when all dates are included.

In sum, throughout the whole cross-section of stocks, we find consistent evidence that the pricing of loan contracts yields lenders significant incremental income that is not due to lenders sacrificing either voting rights or cash flow rights.

Inventory utilization. As a last step, to show the presence of market power, it is essential to also examine lending quantities, in particular, the availability of excess inventory. If

[8]Regulation T mandates that to protect lenders from default, agents borrowing shares have to post cash collateral in an amount equal to 102% of the value of the borrowed securities (typically cash or U.S. Government securities) plus an additional 50% in the form of other collateral. The collateral is marked to market daily.

[9]The result that voting record dates have quantitatively little impact on the share lending market is also consistent with the existing literature, such as Christoffersen et al. (2007) and Aggarwal et al. (2015).

lending fees are set such that lenders obtain income in excess of marginal costs despite the presence of excess inventory, this identifies the role of strategic pricing in the securities lending market. In Panels A and B of Table 4 we document different measures of inventory utilization, including fee-income-weighted inventory utilization. This measure overweights the inventory utilization of stock-date observations with high fee income within each of the 25 stock groups (see the table caption for details), thereby ensuring that the numbers are representative of inventory utilization when prices (fees) are elevated. Independent of the measure considered, we find that across all 25 stock groups, there is significant spare inventory, that is, utilization is significantly below 100%, even among the stocks that generate the highest fees. We conclude that empirically, a lack of spare inventory is generally not the culprit causing deviations from break-even fees. Motivated by these empirical facts, we proceed to developing a dynamic model of shares lending that sheds light on the role of market power in the securities lending market.

3 Model

Four types of financial institutions interact in a dynamic trading environment that features both a centralized limit order market and a securities lending market: (1) *hedge funds* that obtain private information about asset payoffs and may choose to short stocks, (2) *liquidity traders* that trade for liquidity reasons, (3) *regular traders* that can buy or sell shares in the centralized limit order market and can lend out their shares, and (4) a *custodian securities lender* that intermediates shares lending for participating asset owners. In Section 5, we consider a structure for the securities lending market where investors interact directly in a centralized setting rather than using the intermediation services of a custodian lender. Figure 1 provides an overview of the environment.

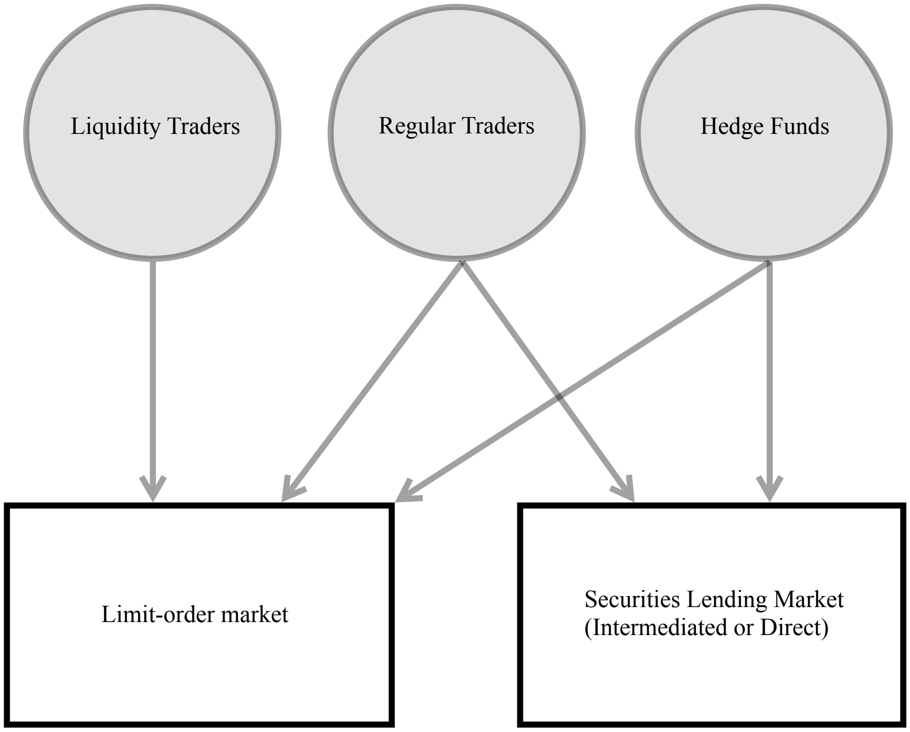


Figure 1: Investor Groups and Markets
The figure illustrates the markets and the different groups of investors in the model. We consider two distinct structures for the securities lending market, one where lending is intermediated via a custodian lender and one where investors interact directly.

Stochastic discount factor. All institutions are firms that are ultimately owned by households who value cash flows according to a stochastic discount factor (SDF) m. This SDF assigns risk premia to aggregate shocks, which are publicly observable. Institutions maximize the present value of their cash flows, given the private information they may have about asset-specific (idiosyncratic) states.[10] Liquidity traders' behavior is further affected by liquidity shocks, as detailed below. The SDF follows the diffusion process:

$$\frac{dm_t}{m_t} = -r_f dt - \chi dB_t, \tag{1}$$

[10]Given the focus of our study, we do not consider conflicts of interest between trading firms and households.

where r_f denotes the risk free rate, B_t is a Brownian motion, and χ is the price of risk for exposures to aggregate shocks dB_t.

Asset. Institutions are trading a generic asset (e.g., a stock) that generates lumpy dividends. Dividends over an interval $[t, t+dt)$ are given by $c_t dN_t$, where N_t is a Poisson process with arrival intensity λ, and where c_t can be interpreted as a productivity-adjusted measure of a firm's capital in place. The time between two innovations to N_t in our model is the stochastic equivalent of what a period would be in a discrete-time setting. However, the stochastic structure we consider has significant advantages in terms of analytical tractability.

Productivity-adjusted capital c_t follows the jump-diffusion process[11]

$$\frac{dc_t}{c_{t-}} = \mu_c dt + \sigma_c dB_t + (e^{v_{t-}} - 1) dN_t, \tag{2}$$

where v_{t-} is a random variable the realization of which is generally private information of some agents, specifically hedge funds. In contrast, the current value of assets in place, c_t, is publicly observable at time t and the aggregate shocks dB_t are unpredictable for all agents. The evolution of the conditional jump size v_t is given by:

$$dv_t = (z_t - v_{t-}) dN_t, \tag{3}$$

where $z_t \sim Normal(-\frac{\sigma_v^2}{2}, \sigma_v)$.[12] That is, the process for v_t stays constant between innovations to the Poisson process N_t and takes a new normally distributed value upon an innovation to N_t. We denote by $f_v(\cdot)$ and $F_v(\cdot)$ the corresponding normal PDF and CDF

[11]In the following, all processes will be right continuous with left limits. Given a process y_t, the notation y_{t-} will denote $\lim_{s \uparrow t} y_s$, whereas y_t denotes $\lim_{s \downarrow t} y_s$.

[12]Specifying $\mathbb{E}[v_t] = -\frac{\sigma_v^2}{2}$ ensures that the idiosyncratic jump term $(e^{v_{t-}} - 1) dN_t$ in equation (2) has zero mean.

of v_t upon such innovations. When quantitatively estimating and evaluating the model in Section 6, we will generalize the dynamics of c_t to capture salient empirical features of the joint dynamics of fee and dividend income.

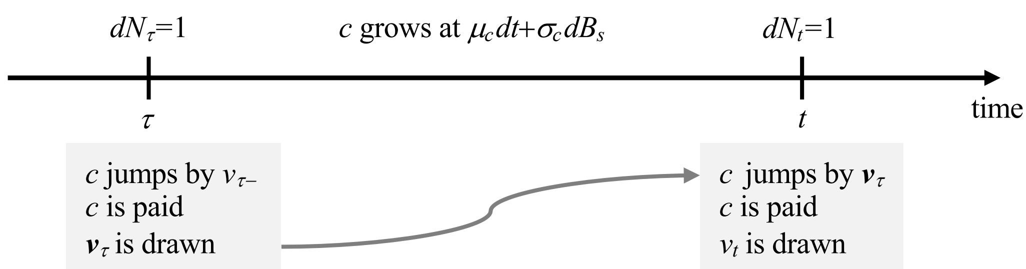


Figure 2: Timing Convention for Growth and Dividends
The figure illustrates the timing of idiosyncratic capital jumps, dividend payments, and changes in the jump size v. Hedge funds have private information about future growth by observing v_τ at time τ.

The timing convention indicated in equation (2) and Figure 2 is important. Consider some date t that features an innovation $dN_t = 1$, as illustrated in Figure 2 (marked on the right-hand side of the graph). At that time, capital c jumps by a log-change equal to v_{t-}, which in turn is still equal to the lagged value v_τ that was realized at the *previous* innovation $dN_\tau = 1$, with $\tau < t$ (marked on the left-hand side of the figure). Logically after capital has been subject to this jump of size v_τ at time t, a dividend equal to the new value of c_t is paid.

While v_τ is not publicly observable at time τ, hedge funds can already observe v_τ at that time and thus have private information about future dividend growth (which in this illustrative example is realized at time t). Once capital has jumped by v_τ at time t, agents can infer what the value of v_τ was by observing this realized change. Yet, at that time, this information has no relevance over and above the new level of c_t, since future growth is independent of v_τ (a new, independent value v_t is drawn at time t).

The Poisson process N_t also governs the frequency with which new hedge funds and cohorts of liquidity traders obtain shocks and can trade in the centralized limit order market at the posted bid and ask prices.

Hedge funds. Upon an innovation to N_t, a new hedge fund arrives to the market and observes the new value of v_t. Each new hedge fund does not have any preexisting holdings of the asset and can, through trading, take positions in the set $\{-\pi, 0, \pi\}$, with $\pi \in (0,1)$.[13] An informed hedge fund becomes a regular uninformed investor after the next shock to N_t. The assumption that the persistence of hedge funds' informational advantage is also governed by shocks to N_t has significant advantages in terms of tractability. While differential information persistence is in principle an interesting feature worth studying, it is not an essential aspect in the context of this paper's objectives.

Liquidity traders. Upon an innovation to N_t, existing liquidity traders owning $(1-\pi)$ units of the asset have to sell their holdings, and a newly arriving cohort of liquidity traders has to purchase $(1-\pi)$ units. The maximum total asset holdings across hedge funds and liquidity traders is thus normalized to one, and the variable π represents a measure of the size of informed hedge funds relative to liquidity traders.

Asset supply. We denote by ρ_0 the quantity of the asset that can in principle be lent out to short sellers or be used to make sales offers in the limit order market. Given that we normalized the total potential asset holdings of hedge funds and liquidity traders to one, the variable ρ_0 represents a relative measure: it is the ratio of the total quantity of the asset that can be lent out or posted at the ask price to the total size of positions taken by hedge funds and liquidity traders.

Conceptually, the total quantity of the asset that is available for lending or sales offers in the limit order market also need not be equal to the total shares outstanding. In practice,

[13] The main results of our analysis are qualitatively unchanged if hedge funds arrive probabilistically conditional on an innovation to N_t.

different classes of investors face legal restrictions and reputational costs to lending out their shares, such as firm managers, founders, and institutional investors do.[14] In our model, such investors follow a buy-and-hold strategy and do not lend out their shares.

Yet consistent with the empirical evidence on inventory utilization below 100% presented in Section 2, we assume throughout that the lendable supply ρ_0 is large enough to ensure shares lending is not mechanically supply constrained, which corresponds to the condition that $\rho_0 > 1+\pi$. Specifically, this assumption ensures that the supply ρ_0 is large enough to avoid mechanical constraints on the following two types of simultaneous activities: (1) The maximum possible demand (excluding hedge funds repurchasing the asset) can be offered for sale at the ask price, which is 1 units of the asset. (2) The maximum possible shorting demand, π units, can be offered in the shares lending market. Note that the units of the asset that hedge funds repurchase and redeliver at the end of a lending contract period become again available as lendable shares at that time. Thus, these units do not take up additional capacity of the supply ρ_0.

We further introduce the variable ρ to denote the units of the asset that are actually posted as lendable inventory in equilibrium (where $\rho \in [0, \rho_0]$). Liquidity traders, which collectively hold $(1-\pi)$ units of the asset at each point in time, do not post their shares in either market to ensure they can sell all their units.[15] Correspondingly, their holdings are not part of the supply of potentially lendable shares ρ_0.

[14] In accordance with the Investment Company Act (1940), open-end funds can lend out at most one-third of their total asset value (see SEC, 1997, 2022). Given that a large fraction of stock market investments are intermediated by mutual funds, this rule substantially reduces the supply for securities lending when compared to the total supply of shares outstanding.

[15] Liquidity traders with positive holdings have to sell their units upon an innovation $dN_t = 1$. Posting a bid quote would not guarantee the sale of all units they wish to sell. Instead, liquidity traders pick up quotes in the limit order book, which ensures a sale with probability 1 in equilibrium. This specification is also consistent with the standard characterization of *liquidity providers* in the literature, which are the ones posting bid and ask quotes, whereas investors picking up those quotes seek liquidity.

Market structure and timing. Upon an innovation $dN_t = 1$, the following logical order of events applies:

1. Dividend income realized at time t is collected by the agents who were the owners of the asset at time $t-$ (that is, t is an ex-dividend date).

2. Investors that wish to lend shares post them with the custodian lender. Posted shares are tied up until settlement (step 7). The custodian lender optimally sets the fee ϕ_t per unit of the asset lent out so as to maximize the expected fee income. Loan agreements start at time t and mature at the time of the next Poisson shock $dN_\tau = 1$ with $\tau > t$.[16] At maturity, a borrower has to return the asset as well as the dividend it pays at that time.

3. A new hedge fund and a new cohort of liquidity investors arrive to the market.

4. The hedge fund can borrow shares at the fee posted by the custodian lender. The custodian lender does not reveal the total demand at this stage.

5. Bid and ask quotes are posted by competitive investors in the centralized limit order market.

6. Investors trade shares at the posted bid and ask prices in the centralized limit order market.

7. Contracts are settled via delivery of the asset in accordance with all lending contracts and trades (steps 2 to 6 above). For example, assets repurchased by a borrowing hedge fund (step 6) are delivered back to the custodian lender, who may pass them on to a new hedge fund as part of a lending contract (agreed to in step 4), who in turn sold them at the bid to an investor (in step 6) and is delivering them to that investor.

[16] Specifying a lending contract with stochastic maturity increases the tractability of the analysis and is analogous to stochastic maturities of debt contracts in many corporate finance models (see, e.g., the extant literature following Leland, 1998).

When a hedge fund observes the new value of v_t, it decides whether to borrow shares at the posted loan fee ϕ_t and to immediately sell these shares at the prevailing bid price, which we denote by B_t. Alternatively, a hedge fund can also purchase shares at the prevailing ask price, which we denote by A_t. Conditional on demanding or supplying shares at a given posted price, traders obtain pro-rata allocations corresponding to the quantity they want to trade at the posted quote. Due to the presence of liquidity traders, investors posting quotes in the centralized limit order market know that these quotes will not just be picked up by privately informed agents, which helps sustain trade in the presence of adverse selection.

4 Analysis

In this section, we analyze the presented baseline model with a delegated securities lending market. We start by characterizing the buy-and-hold value of the asset from the perspective of a regular trader who does not receive liquidity shocks or private signals. Afterwards, we describe how bid and ask quotes are set in the competitive limit order market, what payoffs hedge funds obtain from borrowing shares, and what loan fees are charged by the custodian lender.

Buy-and-hold value. The buy-and-hold value of the asset to an investor who consistently posts shares with the custodian lender is given by the present value of future dividends and fee income. Let P_t be defined as this buy-and-hold-value of one unit of the asset given investors' prior beliefs about the distribution of the current value of v_t at time t, that is, $v_t \sim Normal(-\frac{\sigma_v^2}{2}, \sigma_v)$. Upon on a shock $dN_t = 1$, a hedge fund obtains private information and borrows π units of the asset to immediately sell them at the posted bid quote whenever v is below some endogenous threshold value v_ϕ, that is, when $v < v_\phi$. We will characterize this threshold value v_ϕ below. The probability with which shorting occurs is therefore $F_v(v_\phi)$, and conditional on shorting, the fee income per unit of shares posted with the custodian lender is $\frac{\pi\phi_\tau}{\rho}$ (the fee income is distributed across the ρ units of posted inventory). The ex-dividend buy-and-hold value at time t is thus given by:

$$P_t = \mathbb{E}_t \int_t^\infty \frac{m_\tau}{m_t}(c_\tau + \underbrace{F_v(v_\phi)\frac{\pi\phi_\tau}{\rho}}_{\text{Expected fees}})dN_\tau. \tag{4}$$

Absent signals about the current value of v, the relevant state variable for an investor's valuation is the level of productivity-adjusted capital c. Correspondingly, we proceed to characterizing the value function $P(c)$ for the buy-and-hold value. Since the lending fee will also be chosen without conditioning information about v (see details below), its equilibrium value will be only a function of c as well, that is, $\phi(c)$.

We conjecture and verify that $P(c)$ and $\phi(c)$ are linear functions of c. Going forward, we denote variables scaled by the expected dividend rate, λc, with a tilde. Further, we denote by $Q(c,v)$ the value of the asset conditional on knowing the current levels of c and v, which is the relevant case for hedge funds that obtain private information about v. The following proposition characterizes the value functions $P(c)$ and $Q(c,v)$.

Proposition 1. *The buy-and-hold value of the asset conditional on the prior beliefs about v is given by:*

$$P(c) = \frac{\lambda \cdot (c + F_v(v_\phi)\frac{\pi\phi(c)}{\rho})}{r_f + \sigma_c\chi - \mu_c}. \tag{5}$$

The buy-and-hold value conditional on observing v is given by:

$$Q(c,v) = e^v \cdot P(c). \tag{6}$$

Proof. See Appendix B. □

The valuation formula (5) exhibits the familiar structure under perpetual growth, whereby the expected cash flows in the numerator (here from dividends and fees) are divided by the difference between the discount rate and the growth rate. Further, equation (6) indicates the tractable feature of our setting that privately-informed agents' valuation is simply equal to uninformed agents' valuation $P(c)$ modulated by the factor e^v.

Next, we turn to characterizing the competitive bid and ask quotes that regular traders post in the limit order market.

Bid quotes. Bid quotes are set accounting for potential adverse selection from hedge funds. A hedge fund that observes v will borrow π units of the asset and sell them at the posted bid price B whenever $v < v_\phi$, where v_ϕ is the above-mentioned endogenous threshold value that will be characterized below. Moreover, the current cohort of liquidity traders sells their $(1-\pi)$ units at the posted bid price B irrespective of the realization of v. Anticipating this potential demand, competitive investors offer 1 unit at the bid quote, which is the maximum total quantity the two groups of traders might wish to sell at a given point in time. Competitive pricing implies that the bid price is the highest price B that satisfies the break-even condition:

$$\underbrace{B \cdot \big(\pi F_v(v_\phi) + (1-\pi)\big)}_{\text{Expected Revenue from Sales}} = \underbrace{P \cdot \big(\pi F_v(v_\phi)\,\mathbb{E}[e^v | v \le v_\phi] + (1-\pi)\big)}_{\text{Expected Value of Shares Sold}}, \tag{7}$$

that is, the bid quote B is set such that a competitive investor posting this price breaks even, accounting for the possibility that the offer is picked up by an informed hedge fund or liquidity traders. The left-hand side of equation (7) represents the expected dollar sales revenue: the bid price B times the expected quantity traded. The right-hand side of the equation represents the expected total value of the shares traded at the bid. Equation (7) can be rearranged to:

$$\frac{B}{P} = \frac{\pi F_v(v_\phi - \sigma_v^2) + (1-\pi)}{\pi F_v(v_\phi) + (1-\pi)}, \tag{8}$$

where we use the fact that normality of v implies the following simple representation for the truncated expectations in equation (7):

$$\mathbb{E}[e^{v_t} | v_t \leq v_\phi] = \frac{F_v(v_\phi - \sigma_v^2)}{F_v(v_\phi)}. \tag{9}$$

Ask quotes. Agents providing liquidity to the market by offering shares for sale at the ask price A cannot lend out their shares at the same time, because they need to be able to deliver on their commitment. Thus, these agents not only have to be compensated for being exposed to the standard adverse selection problem but also for forgoing fee revenue in the period where they provide an ask quote. Moreover, ask quotes depend on whether existing shares lending contracts mature at the time of trade or not, as purchases motivated by the delivery of borrowed shares have a different informational content than other trades do. We denote by A_0 the ask price that prevails when current short interest is equal to zero and by A_1 the ask price when short interest is positive and equal to π in equilibrium.

If no shares lending contracts were outstanding at time $t-$, the ask price is the highest price A_0 that satisfies the break-even condition:

$$\underbrace{A_0 \cdot \left(\pi(1 - F_v(\log\frac{A_0}{P})) + 1 - \pi\right)}_{\text{Expected Payments from Purchases}}$$
$$= \underbrace{P \cdot \left(\pi(1 - F_v(\log\frac{A_0}{P}))\,\mathbb{E}[e^v | v \geq \log\frac{A_0}{P}] + 1 - \pi\right)}_{\text{Expected Value of Shares Bought}} + \underbrace{F_v(v_\phi)\frac{\pi\phi}{\rho}}_{\text{Expected Fees Forgone}}. \tag{10}$$

The left-hand side of equation (10) reflects the expected income from sales at the ask price A_0. The right-hand side of equation (10) represents the expected total value that agents posting ask quotes give up: the expected value of the shares traded and the expected fee income from lending out shares. Rearranging equation (10) yields:

$$\frac{A_0}{P} = \frac{\pi F_v(-\log\frac{A_0}{P}) + 1 - \pi + F_v(v_\phi)\frac{\pi\phi}{\rho P}}{\pi(1 - F_v(\log\frac{A_0}{P})) + 1 - \pi}, \tag{11}$$

where we use the fact that normality of v implies the following formula for the truncated expectations in equation (10):

$$\mathbb{E}[e^{v_t}|v \geq \log(A_0/P)] = \frac{F_v(-\log\frac{A_0}{P})}{1 - F_v(\log\frac{A_0}{P})}. \tag{12}$$

In contrast, if there was positive short interest at time $t-$, borrowing hedge funds have to purchase shares to deliver them back to the custodian lender upon a realization $dN_t = 1$. If hedge funds have borrowed, short interest is π units. In this case, the depth of the ask quote is $(1+\pi)$ units, so that both these repurchases and additional purchases from hedge funds and liquidity traders can be accommodated. Correspondingly, the per-unit ask price is the highest value of A_1 that satisfies the break-even condition:

$$\frac{A_1}{P} = \frac{\pi F_v(-\log\frac{A_1}{P}) + \pi + (1-\pi) + (1+\pi)F_v(v_\phi)\frac{\pi\phi}{\rho P}}{\pi + (1-\pi) + \pi(1 - F_v(\log\frac{A_1}{P}))}. \tag{13}$$

Equation (13) reflects the fact that there are now three types of traders potentially picking up the ask quote: (1) A new hedge fund that may have received sufficiently positive information about the asset and wishes to purchase π units, (2) a hedge fund that has to purchase π units to return borrowed assets to the custodian lender, and (3) a new cohort of liquidity traders that needs to buy $(1-\pi)$ units.

It is helpful to clarify why the relations pinning down the ask prices A_0 and A_1 (see equations (11) and (13)) incorporate a term related to fee income, whereas the relation determining the bid price B (see equation (8)) does not. A competitive investor posting an ask price must not have posted her shares with the custodian lender, because shares posted with the custodian are tied up until settlement (see step 7 in the setup description of the market structure and timing). Thus, a regular competitive investor intending to provide liquidity by posting an ask quote has to potentially forgo not only all future dividends and fee income (conditional on selling the asset) but also the potential lending fee income for the upcoming lending contract period. Next, consider the bid quote. An investor intending to provide liquidity by posting a bid quote cannot obtain lending fee income for the upcoming lending contract period if the bid quote is picked up. This is because any shares intended for lending much have been posted with the custodian lender in advance and must be available with probability one. Thus, since any shares that will be purchased at the bid do not deliver fees for the upcoming contract period, the relation for the bid price (8) does not reflect such fees.

Hedge funds' borrowing decisions. A hedge fund observing v optimally borrows whenever the value derived from borrowing shares exceeds the loan fee, that is

$$B(c) - L(c,v) \geq \phi(c), \tag{14}$$

where B is the bid price at which the hedge fund sells the shares immediately after borrowing the shares (in step 6) and $L(v,c)$ denotes the present value of the liability of having to return the shares and the dividends they pay at the end of the loan period.[17] The following proposition characterizes the value of this liability.

[17] Recall that trades in the limit order market are ex dividend. Thus, a hedge fund purchasing shares at time t does not get a claim to the dividends those shares pay at time t. However, the lender requires to receive the dividends that will be paid at the next innovation of N_t.

Proposition 2. *A hedge fund with an open short position has a liability with the following present value*

$$L(c,v) = e^{v} \frac{\lambda \cdot (c + A_1(c))}{r_f + \sigma_c \chi - \mu_c + \lambda}. \tag{15}$$

Proof. See Appendix B. □

The solution to the value of a hedge fund's liability (15) reflects the fact that, upon maturity of the lending contract, the fund has to both provide the dividend c and repurchase the shares at the then prevailing ask price A_1. Moreover, equation (15) indicates that hedge funds observing negative information (lower future growth v) anticipate that it will be cheaper to fulfill this liability, thus making it more profitable to borrow.

Optimal loan fee. As a last step, we derive the optimal loan fee a custodian lender charges in order to maximize the value of fee income received by its clients. The lender solves a classic monopolistic screening problem, since it does not know the hedge fund's private information (type) when posting the fee.[18] When choosing the optimal loan fee, the custodian lender faces a trade off between the probability of lending and the fee collected conditional on lending. Choosing a loan fee ϕ is equivalent to pinning down a marginal hedge fund type v_ϕ that is just indifferent between borrowing and not borrowing at the posted fee.

Proposition 3. *The custodian lender's optimal loan fee is equal to the marginal hedge fund type's value from having borrowed shares, that is,*

$$\phi(c) = B(c) - L(c, v_\phi), \tag{16}$$

[18]This is the optimal mechanism from the perspective of investors posting the inventory of lendable shares when facing a privately informed hedge fund. See also Holmstrom and Myerson (1983) for the implementation of durable decision rules. Note that the equilibrium in our model is Pareto efficient.

where the marginal type v_ϕ solves the equation:

$$\frac{f_v(v_\phi)}{F_v(v_\phi)} \cdot \left(\frac{B(c)}{L(c, v_\phi)} - 1\right) = 1. \tag{17}$$

Proof. See Appendix B. □

The reversed hazard rate $\frac{f_v(v_\phi)}{F_v(v_\phi)}$ in equation (17) reflects the typical marginal trade off associated with screening problems: marginally raising fees has the cost of excluding the marginal type (which has density $f_v(v_\phi)$) and the benefit of collecting higher revenue from all inframarginal types (which have mass $F_v(v_\phi)$).

Discussion: loan fees and abnormal returns. Our model predicts that if econometricians disregard loan fees when computing returns to investors, they find negative abnormal returns in asset pricing tests, especially for assets with high fee income. This result is consistent with empirical evidence that asset pricing anomalies are concentrated in the short leg, among high-fee stocks and are often insignificant once fees are incorporated (see in particular Stambaugh et al., 2011; Muravyev et al., 2022).

In the context of our model, we define the following measure:

$$\text{fee yield} \equiv \frac{\lambda F_v(v_\phi)\frac{\pi\phi}{\rho}}{P}, \tag{18}$$

which is the analogue of the dividend yield but considers investors income from shares lending. Note that this additional income is collected by active stock owners in our model that either make their shares available for lending or offer liquidity by posting ask prices. If an econometrician chooses the correct SDF but implements standard asset pricing tests that only account for dividends and capital gains when computing returns, she will on average estimate the following buy-and-hold alpha:

$$\alpha = -\text{ fee yield.} \tag{19}$$

That is, the estimated alpha is equal to the negative of the fee yield. Given this alpha, we can express the buy-and-hold value as follows:

$$P(c) = \frac{\lambda \cdot c}{\underbrace{r_f + \sigma_c \chi + \alpha}_{\text{Average Return}} - \mu_c}. \tag{20}$$

Another context in which alphas would be detected empirically is when econometricians use benchmark indices to measure return performance. Suppose that the asset's cash flow c represents the cash flows generated from the firms in such an index. Then, relative to the performance of the index that excludes lending fee income, a shares lender's portfolio outperforms by $\alpha =$ fee yield, that is, a *positive* expected excess return is estimated.

In sum, standard asset pricing tests will detect abnormal returns and the magnitudes of these returns are predicted to increase with (1) loan fees and (2) the quantity of shares lent out, but decreasing in the quantity posted with the custodian lender. Thus, if fewer shares are available for lending, abnormal returns are predicted to increase. Loan fees, in turn, are predicted to increase with information asymmetry, e.g., when bid-ask spread income is larger (bid-ask spreads times volume traded). Moreover, the quantity of borrowing should be related to information asymmetry and hedge funds' relative trading capacity (as measured by $\frac{\pi}{\rho}$).

5 Delegated vs. Centralized Securities Lending

To provide insight on the implications of shares lending being delegated to a strategic custodian lender, we analyze in this section how equilibrium outcomes would differ if the shares lending market were instead centralized and competitive. To do so, we consider the most generic form of a competitive securities lending market, that is, a market that mirrors the prevalent limit order markets that are used for regular securities trading in practice: suppliers of lendable shares submit competing offers for loan contracts and hedge funds can pick the best offer. We further discuss additional market structures in Section 7.

Contrasting the market structures reveals the key economic features of delegation that is present in our baseline model: in this baseline setting, the custodian lender strategically chooses fees so as to maximize the value for investors lending their shares, and it does not instantaneously reveal shorting demand to all market participants (see step 4 of the description of the baseline setup). In contrast, such secrecy is absent in the centralized shares lending market, which potentially harms hedge funds' ability to extract rents from negative private information due to information leakages.

In the setting with a centralized lending market, the logical order of events upon an innovation $dN_t = 1$ is generally identical to the one we laid out for the delegated market, with differences only pertaining to steps 2 and 4. Specifically, in step 2, individual investors directly quote lending contract fees (rather than posting their shares with a custodian lender). In step 4, these posted offers can be taken up by hedge funds, with realized lending volume being publicly observable. We reiterate the detailed description of the logical order of events under this market structure in Appendix Section A.

The following proposition describes key equilibrium outcomes when shares lending is moved to this centralized market arrangement.

Proposition 4. *When shares lending is moved to the centralized market, the following* *equilibrium outcomes obtain:*

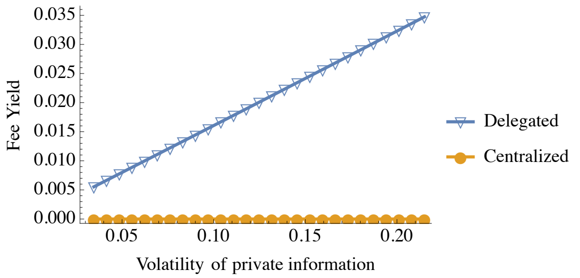
- *Shares lenders are strictly worse off if the shares lending market is centralized, since the lending fee is zero, that is,* $\phi = 0$.
- *Hedge funds may be worse off if the shares lending market is centralized, in particular if hedge funds' private information* (σ_v) *or their relative trading capacity* (π) *is sufficiently large.*

Proof. See Appendix B.4. □

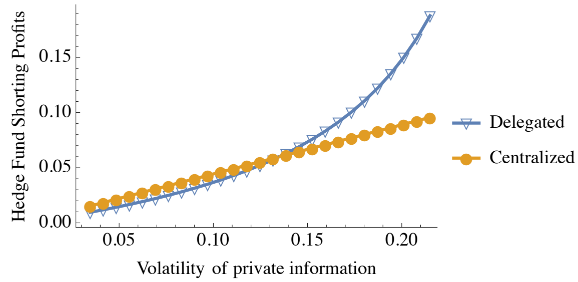
Proposition 4 confirms that in the centralized lending market, lending fees are competed down to zero, which unambiguously harms shares lenders. Who are ultimately these shares lenders in practice? While large asset management companies act as custodian lenders in today's markets, they pass on large fractions of their securities lending income to fundholders. For example, Vanguard passes on 98.2% of its gross securities lending revenue from U.S. equities to their fundholders (see Morningstar, 2018). Thus, retail and pension investors would be negatively affected by the elimination of lending fee income.

Moreover, the proposition reveals that the centralized shares lending market may limit hedge funds' ability to profit from negative private information, in particular when the underlying security is illiquid (which occurs when hedge funds are more informed and have relatively more trading capacity). This is because information leakages associated with a centralized securities lending market are more consequential when securities are relatively less liquid to begin with.

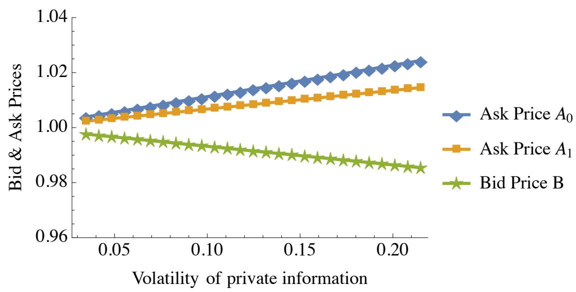
Figure 3 illustrates these results with comparative statics. Panel (a) compares the fee yield lenders obtain conditional on shares lending being centralized versus delegated to a custodian lender. Whereas competition drives down fees to zero in the centralized securities lending market, fee yields are positive and increasing as a function of hedge funds' private information.



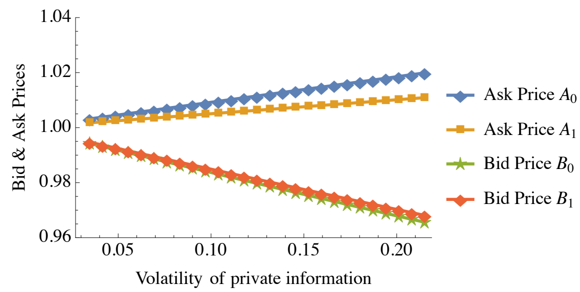
(a) Fee yield across markets



(b) Hedge fund shorting profits across markets



(c) Limit orders with delegated securities lending



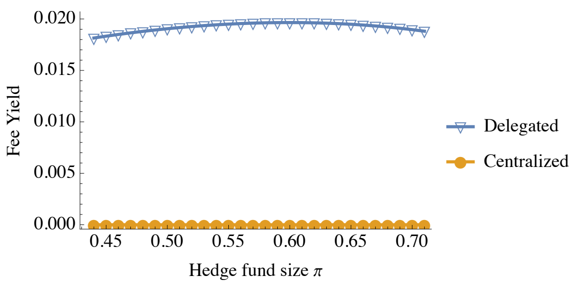
(d) Limit orders with centralized securities lending

Figure 3: The figure compares market outcomes for the cases of delegated securities lending and centralized securities lending. On the horizontal axis of each graph, we vary the level of the volatility of hedge funds' private information, $(\lambda \cdot (e^{\sigma_v^2} - 1))^{1/2}$, by adjusting the parameter σ_v. Panels (a) and (b) plot fee yields and expected shorting profits of hedge funds. Panels (c) and (d) plot limit order market quotes in the delegated and centralized market, respectively. The subscripts on the quotes distinguish states with and without preexisting short interest (denoted as 1 and 0, respectively). The bid prices indicated in panel (d) are conditional on observing new securities borrowing, but differ by whether there is preexisting short interest or not. The parameters are given by $\lambda = 12$, $\pi = 0.55$, $\rho_0 = 2.1$, $\mu_c = 0.042$, $\sigma_c = 0.2$, $\chi = 0.4$, and $r_f = 0.01$.

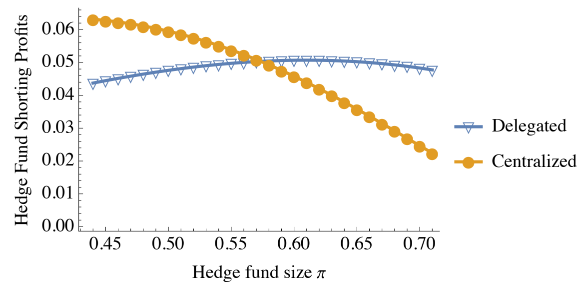
Panel (b) compares informed hedge funds' profits from shorting across the two market structures. Whereas informed shorting is more profitable in the centralized securities lending market when hedge funds have less private information, delegated shares lending is preferable for them when they are more informed. When comparing market structures from the perspective of hedge funds, there is a tradeoff between lending fees (paid in the first step of establishing a short position) and the price obtained from selling immediately after borrowing shares, as represented by the bid quotes. While hedge funds do benefit from paying zero borrowing fees in the centralized securities lending market, they are harmed by information leakages and the associated repricing of securities.

This effect is illustrated in the bottom two panels of Figure 3. Panel (c) plots bid and ask quotes in the limit order market when securities lending is organized as a delegated market structure, and Panel (d) considers the case where securities lending is centralized and provides quotes conditional on borrowing having occurred in the securities lending market. In the case of centralized lending, hedge funds follow a mixed strategy whereby they borrow securities even when they have observed good news (this also leads the bid prices to be conditional on preexisting short interest, whereas there is only one bid price in the delegated market structure). However, despite this mixed strategy, bid prices are lowered conditional on shares borrowing having occurred, which harms hedge funds' profits from shorting. Indeed, Panel (d) shows that the conditional bid prices offered in a centralized securities lending market are substantially lower than the ones offered when shares lending is delegated (Panel (c)), in particular when market participants know that hedge funds have more private information.

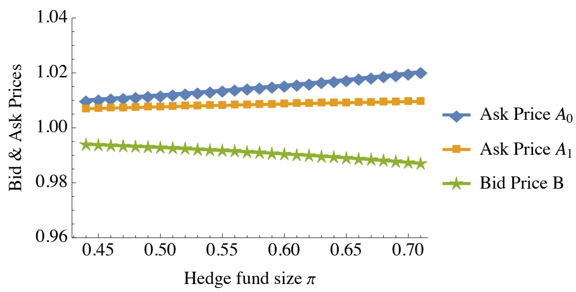
Figure 4 illustrates that variation in the parameter π, which governs the relative size of the positions that hedge funds can take, has somewhat similar implications. The panels of this figure follow the same structure as the ones of Figure 3. Note that since the sum of the maximum holdings of hedge funds (π) and liquidity traders ($1-\pi$) is normalized to one, increasing π not only raises hedge funds' trading capacity, but also decreases liquidity trading. As a result, a higher level of π reduces liquidity and can potentially reduce fee yields (see Panel (a)) and hedge fund profits (see Panel (b)). Panel (b) shows the key result that for values of π exceeding a threshold, hedge fund profits from shorting are again higher under the delegated market structure. As in the case of high values of σ_v, the market is less liquid for higher values of π, implying that bid prices respond more strongly to information leakages from the securities lending market. Therefore, centralizing securities lending can



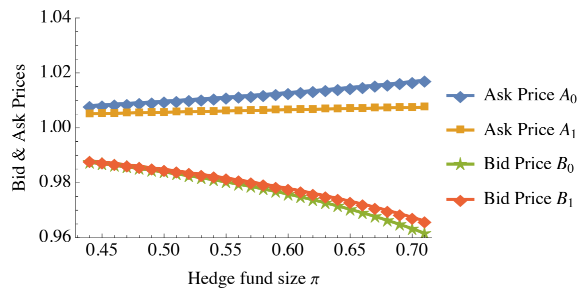
(a) Fee yield across markets



(b) Hedge fund shorting profits across markets



(c) Limit orders with delegated securities lending



(d) Limit orders with centralized securities lending

Figure 4: The figure compares market outcomes for the cases of delegated securities lending and centralized securities lending. On the horizontal axis of each graph, we vary the relative size of the positions that hedge funds can take, π. Panels (a) and (b) plot fee yields and expected shorting profits of hedge funds. Panels (c) and (d) plot limit order market quotes in the delegated and centralized market, respectively. The subscripts on the quotes distinguish states with and without preexisting short interest (denoted as 1 and 0, respectively). The bid prices indicated in panel (d) are conditional on observing new borrowing, but differ by whether there is preexisting short interest or not. The parameters are given by $\lambda = 12$, $\sigma_v = 0.035$, $\rho_0 = 2.1$, $\mu_c = 0.042$, $\sigma_c = 0.2$, $\chi = 0.4$, and $r_f = 0.01$.

ultimately erode hedge fund profits from shorting.

Overall, these results yield the prediction that informed agents borrowing securities tend to prefer the delegated market structure for more illiquid securities, such as small stocks in practice. Moreover, investors lending their shares consistently prefer the delegated market structure.

6 Quantifying the Value Implications of Short Selling

In this section, we proceed to addressing the key quantitative questions of our paper: what is the impact of non-competitive fees on security lenders' stock valuations and what is the distribution of alphas that short sellers obtain from targeting different segments of the cross-section of stocks?

Our solution for the price-dividend ratio (5) provides some initial intuition for the impact of fee revenues on security lenders' valuations. Let ψ denote the fee-to-dividend ratio. Rearranging (5) we obtain the following relationship between the price-dividend ratio that capitalizes fee income and the one that does not:

$$\text{P-D ratio with fee income} = \text{P-D ratio w/o fee income} \cdot (1+\psi)\,. \tag{21}$$

Equation (21) reveals that the fee-to-dividend ratio ψ is an essential statistic determining the price level effect of lending fee income and in fact, in our baseline model, ψ exactly measures this price-level effect, which we will also refer to as a *value wedge*.[19] Yet, more generally, the dynamic properties of fee income have to be recognized to accurately quantify these price-level effects in practice. After all, the less persistent lending fees are, the less does the current fee-to-dividend ratio matter. Consequently, as a first step, we now

[19]This value wedge is closely related to the concept of the price wedges in van Binsbergen et al. (2023), which measure deviations of market prices from fundamental values.

examine the dynamic properties of fee income in the data.

6.1 Empirical Fee-Yield Dynamics

For our analysis, we are interested in both the cross-sectional and the time-series properties of lending fee income. Table 5 sheds light on the cross-sectional dispersion of lending fee income by revisiting our 25 groups of firms split based on size and fee yields. The table reveals large cross-sectional dispersion in fee yields for each size group, with a highly non-linear relation between fee yields and fee yield percentiles. This non-linearity motivated our choices for the percentile cutoffs. For the large-cap stocks in the Russell Top 200, the bottom 80% have an average fee yield less than 0.01%. In contrast, the top 2% have an average fee yield of 0.81%. Moreover, market capitalization has significant implications for fee yields: for the nano-cap stocks which are not included in the Russell indexes, the top 2% of stocks generate an average fee yield of 63.65%. For the small-cap and micro-cap stocks, this number is 8.51% and 28.96%, respectively. Table 6 further reveals significant differences in dividend yields across firms in the 25 groups (see "Data" columns) with a negative correlation between dividend yields and fee yields applying both in the fee and the size dimensions of our sorts.

If fee income and dividend income were perfectly persistent, we could immediately determine the price level implications of fees by computing the fee-to-dividend ratio, as shown in equation (21). However, as illustrated by the Markov transition matrix reported in Table 7, firms do migrate consistently across the 25 groups and correspondingly exhibit non-trivial fee yield dynamics. To gauge the pricing implications of fee income, it is therefore essential to account for the persistence of fee yields and their joint dynamics with dividend yields, which will be confirmed by our quantitative estimates. The Markov matrix reveals that the association with a given size group is highly persistent whereas fee group association is persistent but less so. Moreover, persistence in fee group association varies materially by size.

Motivated by these observations, we proceed with generalizing our baseline model in order to account for these empirical dynamics. Equipped with this estimated model, we can then proceed to evaluating the key quantitative questions of this paper: how does market power in the shares lending market affect cross-sectional stock valuations and what is the distribution of alphas that short sellers obtain from targeting different segments of stocks?

6.2 Generalizing the Model Dynamics

Corresponding to the 25 bins in our empirical analysis, we consider a 25-state Markov chain indexed by the state $s \in \Omega = \{1,...,25\}$. We denote the 25×25 Markov Generator matrix by Λ and by $M_t(s,s')$ a counting process keeping track of jumps from state s to s'. The capital evolution equation (previously (2)) now takes the form:

$$\frac{dc_t}{c_{t-}} = \mu_c dt + \sigma_c(s_{t-})dB_t + (e^{v_{t-}} - 1)dN_t + \sum_{s' \in \Omega} \left(e^{u(s') - u(s_{t-})} - 1 \right) dM_t(s_{t-}, s'), \tag{22}$$

where the exposure to aggregate risk $\sigma_c(s)$ and the volatility of private information, $\sigma_v(s)$, can now vary with the Markov state s. Moreover, the generalized dynamics (22) allow for capital c to jump by a log change $(u(s') - u(s))$ when the Markov state changes from s to s'. This implies that firms' expected dividend growth rates vary with the state s, taking the form:

$$\mu_c + \sum_{s' \in \Omega} \left(e^{u(s') - u(s_{t-})} - 1 \right) \Lambda(s_{t-}, s'). \tag{23}$$

When considering a cross-section of a continuum of firms, this specification implies the desirable feature of a stationary size distribution around a common trend with growth rate μ_c.

For the purpose of determining valuation level effects of fee income, the key empirical metric regarding the shares lending market is the fee-to-dividend ratio $\psi(s)$ in each state s. Our estimation directly pins down this ratio using the empirical counterpart: for each bin s, it is the ratio of the fee yield (Table 5) to the dividend yield (Table 6).

As we will show below, this generalized version of our model can capture the empirical relation between dividend yields and fee yields both in terms of the cross-sectional distribution across the 25 bins and in terms of the persistence in a firm's assignment to any given bin (as captured by the Markov matrix in Table 7).

Defining value wedges. To determine the effect of lending fees on the level of equity valuations, we analyze the relation between two distinct present values:

$$P_t = \mathbb{E}\left[\int_t^{\infty} \frac{m_\tau}{m_t} c_\tau \cdot (1+\psi(s_t))\, dN_\tau\right]. \tag{24}$$

$$P_t^e = \mathbb{E}\left[\int_t^{\infty} \frac{m_\tau}{m_t} c_\tau dN_\tau\right]. \tag{25}$$

Whereas the valuation equation (24) capitalizes fee income, the valuation equation (25) does not. The valuation effect of fee income is then measured by the *value wedge*:

$$VW(s) \equiv \frac{P_t}{P_t^e} - 1. \tag{26}$$

This measure reflects value inflations in the sense that it captures the extra value that shares generate for shareholders due to lending fee income rather than due to firms' fundamental real productivity and associated dividend income. This value wedge does not, however, reflect irrationality or mispricing. Rather, it emerges as a result of the interaction between information asymmetry and market power in the shares lending market.

HJB equations and closed-form solutions. Valuations again scale with the level of capital c, yet now they also depend on the Markov state s. Let $\mathbf{P}(c)$ and $\mathbf{P}^e(c)$ denote the 25×1 vectors collecting the valuations $P(c,s)$ and $P^e(c,s)$ and let $\Lambda(s)$ be the s-th row of the Generator matrix Λ. Further, let $\mathbf{U}(s)$ denote a row vector collecting the values of $e^{(u(s')-u(s))}$ conditional on being in state s and ψ the vector of fee-to-dividend ratios. We obtain the following closed-form solutions for valuations with and without fee income.

Proposition 5. *The vectors of valuations with and without fee income have the solutions:*

$$\mathbf{P}(c) = \lambda c \cdot (\Lambda \odot \mathbf{U} - diag(r_f + \sigma_c(s)\chi - \mu_c(s)))^{-1}(\mathbf{1} + \psi), \tag{27}$$

$$\mathbf{P}^e(c) = \lambda c \cdot (\Lambda \odot \mathbf{U} - diag(r_f + \sigma_c(s)\chi - \mu_c(s)))^{-1}\mathbf{1}, \tag{28}$$

where $\odot$ denotes the Hadamard product.

Proof. See Appendix B. □

6.3 Estimating the Model

In this section, we discuss our estimation approach by detailing the moments in the data that identify the model's parameters. The model estimation has two main objectives related to shorting activity: to quantify (1) the surplus accruing to shares lenders and (2) the surplus internalized by short sellers. Corresponding to these two objectives our model estimation follows a modular approach. In a first step, we estimate the model taking observed fees as given. This approach is feasible since the joint dynamics of dividends and fee income are a sufficient statistic for computing the value added accruing to shares lenders. The key moments targeted by this estimation are listed in Tables 5, 6, and 8. In a second step, discussed in Section 6.5, we expand the model estimation to structural parameters that pin down endogenous fee dynamics and trading gains for short sellers in the model (in particular those relating to private information).

SDF parameters. We choose a price of aggregate risk $\chi = 0.40$, which is identified by an equity premium of 8% and an equity market volatility of 20%. We pick r_f to match the historical average short-term rate of 4.3%.

Risk exposures. We directly match expected return estimates (not incorporating fee income) for firms in each bin as implied by the CAPM, which pins down the risk exposure parameters $\sigma_c(s)$ for each state s. In addition, we perform robustness tests using the Cahart 4-Factor model. The values of the risk premia across the 25 bins for these two models are listed in Table 8.

Fee-to-dividend ratios. The fee-to-dividend parameters $\psi(s)$ are chosen directly to match the ratio of the fee yield (Table 5) to the dividend yield (Table 6) in each state. We will estimate the deep parameters determining these fee-to-dividend ratios in equilibrium in Section 6.5.

Dividend dynamics. We choose the dividend trend growth parameter μ_c to match the general level of price-dividend ratios across all states and estimate the dividend growth process parameters $u(s)$ by targeting the dividend yields in all 25 bins. As shown in the "Model" columns of Table 6, the model-implied dividend yields $(1/\tilde{P}(s))$ closely match their data counterparts in all 25 states. We directly estimate the Generator matrix Λ by converting the monthly Markov transition matrix reported in Table 7 into its continuous-time counterpart.

6.4 Value Added from Securities Lending

Equipped with the estimated model, we proceed with evaluating our first set of key quantitative questions: how does non-competitive fee income affect the value of shares in the cross-section and at the aggregate stock market level?

State-contingent value wedges $VW(s)$. Figure 5 plots the value wedges $VW(s)$ in all 25 states. To reiterate, these value wedges measure the incremental value that shares lenders assign to stocks, due to the fact that they collect lending fee income. If shares lenders are also marginal investors in these stocks, then realized transaction prices do reflect this additional value. The value wedges range from 1.50% for the large-cap stocks with the lowest fees (bottom 80%) to 100% for the group of highest-fee stocks with the smallest market capitalization (i.e., those not included in the Russell Indexes).

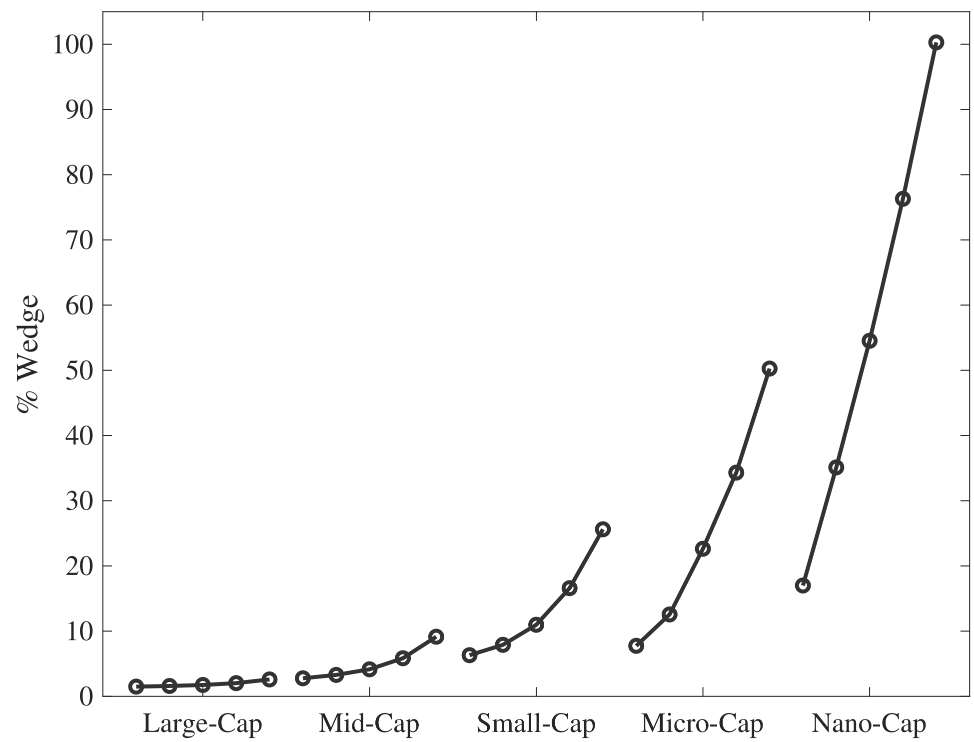


Figure 5: Value Wedges VW due to Lending Fee Income
We plot the additional percentage value that shares lenders assign to stocks due to lending fee income. If shares lenders are marginal investors, these are equilibrium value wedges over and above the prices that reflect the productive value of a firm. Due to securities lending, shares have a dual role as reflecting firm fundamentals and informed agents' information rents that are internalized by shares lenders via fees. See Panel A of Table 9 for the numbers plotted in this graph.

These results differ dramatically from those one would obtain when ignoring fee and dividend dynamics: if fee-to-dividend ratios did not exhibit time variation for firms in each given state, the value wedges would instead range from 0.15% to 60142%. Accounting for fee dynamics is thus of first-order importance for determining accurate value wedge estimates. Despite the significant persistence of group association indicated in the Markov matrix reported in Table 7, the documented transition rates play a critical role. While there is a 96% chance that a Large-Cap firm (i.e. a Russell Top 200 member) in the lowest fee yield group remains in that group, there is a 3% chance of transitioning to the next-higher fee yield group in the Large-Cap group, which is quantitatively important. Valuations in our estimated model account for these transitions and therefore lead to significant value wedges even for those Large-Cap stocks that in their current state do not have high fee yields.

Normatively, these results imply that at the margin, shareholders not participating in shares lending miss out on a significant fraction of value, in particular when investing in small stocks. While the results in Figure 5 show the value added that shareholders obtain that actually participate in shares lending, one can also consider the following hypothetical: suppose fee income were distributed equally across all shareholders, even those that are not participating in shares lending. In this case fee yields are mechanically lower, since the same lending fee income is split across more shares. Panel B of Table 9 reports the value wedges for this counterfactual scenario. The value wedges then vary between 0.49% and 10.69%, which is still economically significant, especially for small stocks.

Aggregate value effects and their cross-sectional distribution. Next, we turn to evaluating the aggregate, market-wide value added due to fee income and the cross-sectional distribution of this incremental value. We find that the total value added from fee income, expressed as a fraction of the total market value of shares in inventory, is 3%. That is, a representative shares lender obtains about 3% more value from its holdings than it would absent shares lending. Panel A of Table 10 further reports the distribution of that total value added across the 25 stock groups. Whereas small-cap high-fee stocks have high value wedges, they represent a relatively small fraction of the aggregate stock market capitalization. This is not surprising given that our highest-fee groups represent only 2% of the stocks in each size category. As a result, from an aggregate perspective, low-fee, large-cap stocks are still an important source of fee income. This fact is recognized by large institutional investment management companies such as Vanguard, which are able to offer particularly competitive fund management fees on index products due to significant securities lending fee income.

Finally, we again perform this evaluation under the counterfactual distribution of fee income whereby all shareholders obtain a pro-rata allocation of the total fee income. In this case, the total value added, expressed as a fraction of the total market value of all outstanding shares (not just the shares posted as inventory) is about 1%. The relative distribution of this value added across the 25 firm groups, which is reported in Panel B of Table 10, is quite similar to the one we obtained when considering only shares posted as inventory. Again, the groups of large-cap and low-fee stocks represent an important source of income at the aggregate level.

6.5 Short Sellers' Net-of-Fee Surplus

Our analyses of value wedges in Section 6.3 did not require estimating or specifying the private-information parameters $\sigma_v(s)$, as well as the parameters π, λ, and $\rho_0(s)$, since empirically observed fee revenues were a sufficient statistic describing lenders' incremental income. In contrast, in order to quantify short sellers' net-of-fee surplus, these model parameters have to be estimated. In the following, we first describe the estimation approach and thereafter discuss the results.

6.5.1 Estimation of Additional Structural Parameters

In this section, we describe our estimation approach for the deep parameters determining endogenous fee dynamics and trading gains for short sellers in the model.

Expected period length. We set $\lambda = 365$, which implies that on average, lending contracts mature after one day, consistent with the fact that security lending contracts in practice are typically overnight contracts. However, we have also performed robustness analyses showing that our shorting surplus estimates are hardly affected when the model is re-estimated conditional on lower values for λ.[20]

Shares available for lending. We set $\rho_0(s)$ in each state such that the utilizations in the model match the state-contingent utilization estimates documented in Table 4.[21]

Hedge funds' private information. The parameters $\sigma_v(s)$ pin down how much private information potential short sellers obtain in each state s about a firm's future performance. Ceteris paribus, higher values of $\sigma_v(s)$ increase potential short sellers' willingness to pay, implying that observed fees, specifically fee-to-dividend ratios, are informative moments for these parameters. In our estimation, we are therefore choosing $\sigma_v(s)$ in each state to exactly match the empirically observed fee-to-dividend ratios, which, as discussed in Section 6.3, are computed as the ratio of the fee yield (see Table 5) to the dividend yield

[20]This result is related to the invariance concepts highlighted by Kyle and Obizhaeva (2017).

[21]Recall that since the sum of hedge funds' and liquidity traders' maximum holdings is normalized to one, $\rho_0(s)$ represents a relative measure: it is the ratio of the total quantity of the asset that can be lent out or posted at the ask price to the total size of positions taken by hedge funds and liquidity traders.

(see Table 6). In the model, the fee-to-dividend ratio is given by:

$$\psi(s) = \frac{\lambda F_{v,s}(v_\phi(s))\frac{\pi\phi(s,c)}{\rho}}{\lambda c}. \tag{29}$$

Hedge funds' relative trading capacity. Hedge funds' relative trading capacity π increases the quantity of privately informed trading in the market unconditionally (across all states s). We set this parameter to $\pi = 0.3$, consistent with the fact that hedge funds, mutual funds, and banks account for approximately 30% of US equity trading volume (see Martin and Wigglesworth, 2021; Mackintosh, 2021).[22] Our results are, however, quantitatively similar when an alternative value for π is chosen and the other parameters are re-estimated conditional on this different value for π.

This robustness obtains since π and σ_v have similar effects in the following sense: if informed traders have a lower trading capacity (low π) but then have a lot of private information (high σ_v), the implications are similar for short sellers' net-of-fee surplus as when informed traders have a larger trading capacity (higher π), but have less private information (lower σ_v).

6.5.2 Estimates of Short Sellers' Alphas and Private Information

We define the annualized expected net-of-fee surplus yield from shorting:

$$\alpha_{short} \equiv \lambda \cdot F_{v,s}(v_\phi(s)) \cdot \left(\underbrace{\frac{B(c,s)}{P(c,s)} - \frac{\mathbb{E}[L(c,v,s)|v < v_\phi(s)]}{P(c,s)}}_{\text{Expected before-fee gain from shorting}} - \underbrace{\frac{\phi(c,s)}{P(c,s)}}_{\text{Fee}} \right). \tag{30}$$

[22] Specifically, we take the sum of trading volumes of mutual funds, traditional hedge funds, quant hedge funds, and banks in 2021 as documented in Martin and Wigglesworth (2021).

The trading strategy achieving this annualized alpha involves shorting the stock when hedge funds do so and having a zero exposure otherwise. That is, achieving this alpha requires hedge funds' private information.

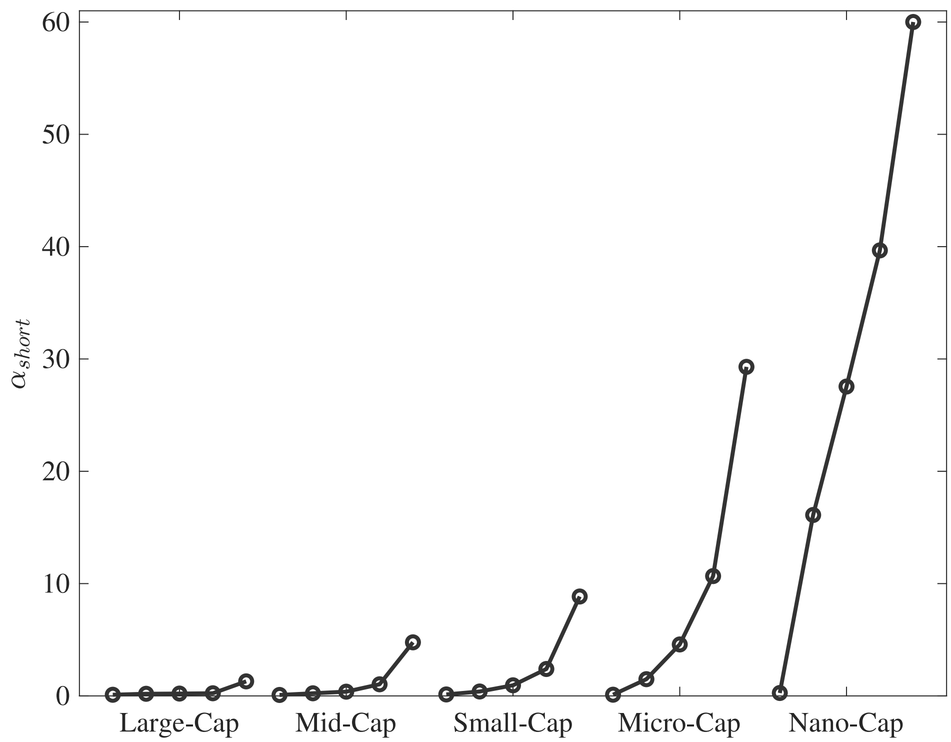


Figure 6: Short Sellers' Alphas (in percent)
The figure plots the annualized expected net-of-fee alphas from shorting across the 25 size and fee-yield groups. The trading strategy achieving this annualized alpha involves shorting the stock when hedge funds do so and having a zero exposure otherwise.

Figure 6 illustrates these model-implied alphas from shorting (see Table 11 for the exact numbers). The values for α_{short} range from 0.1% per annum for the group of the largest low-fee stocks to 60% for the nano-cap high-fee stocks. For small-cap stocks the highest fee group yields an alpha from shorting equal to 8.9% per annum. Across states, fees account for about 56% of the before-fee surplus (see the terms indicated in equation (30)). That is, fees substantially reduce short sellers net-of-fee abnormal returns α_{short}.

What are the magnitudes of private information hedge funds obtain in order to generate these abnormal returns? Table 12 documents the annualized volatilities of idiosyncratic shocks to firms' returns that are privately known by hedge funds according to our estimates. These volatilities range between 0.1% and 36%. Note that, as expected, these estimates are all well below the levels of idiosyncratic return volatility that each of these groups exhibit, since not all idiosyncratic variation in prices is due to privately informed trades (see Table 13 for estimates of idiosyncratic volatility). However, there is substantial variation across groups. In particular, in high-fee states, the fraction of volatility that is private information is substantially higher.

7 Discussions

In this section, we discuss the implications of alternative market designs and the coexistence of options markets.

7.1 Alternative Market Structures

In Section 5, we considered the most generic form of a competitive securities lending market, specifically a market that mirrors the prevalent limit order markets that are used for regular securities trading in practice: suppliers of lendable shares submitted their offers for loan contracts and market participants interested in borrowing shares competed for those offers. We highlighted that this centralized market structure leads to information leakage and can thereby harms short sellers' ability to profit from borrowing securities.

Alternatively, one might envision a novel market structure that is competitive while aiming to mitigate information leakage effects. Specifically, such a market structure for securities lending would need to avoid the public display of realized trades for a certain pre-specified time period, while at the same time allowing for the delivery of shares and clearing. Agents offering their shares for lending would post them with a platform and would not obtain information about whether their shares were actually lent out until the end of the trading day (in the context of our model until clearing step 7). Only agents attempting to borrow would receive information about whether they were successful prior to the end of the day, which is necessary to ensure that those agents know whether they can sell securities. However, upon settlement of those trades, parties would have to be informed about whether their shares were lent out or not, at which point information would leak to other market participants.

It is useful to discuss other practical implications and potential shortcomings of such an alternative market structure. First, while this market structure would benefit hedge funds, it would harm investors lending shares, which in practice are effectively largely retail investors. This is because large asset management companies that act as custodian lenders pass on substantial fractions of their securities lending income to their investors. As noted above, according to Morningstar (2018), Vanguard, which is a major player in the securities lending market, passes on 98.2% of its gross securities lending revenue from U.S. equities to their fundholders, which are regular investors and pension investors. Moving to a competitive but intransparent securities lending market would move surplus from these regular investors to hedge funds, which may or may not be desirable from regulators' and policy makers' perspective.

Second, implementing such a novel market structure might face additional complications when short sellers follow involved dynamic trading strategies that require secrecy for extended time periods of potentially uncertain length. That is, a mechanical one-size-fits-all schedule of revealing securities lending volume at a daily frequency might still harm those short sellers' profits, causing them to potentially prefer delegation to an intermediary that hides realized borrowing volume for longer time periods. Indeed, in our data, short sellers maintain their positions for about 40 days on average, and there is large variation around this average.

Finally, commitment to secrecy by platforms might be a more significant issue when security lending is competitive and does not generate profits. In the current regime, custodian lenders do benefit from continued reputation regarding their commitment to facilitate secrecy as securities lending yields fee income and the ability to have more competitive investment products that offer fund investors lower fees. For example, Vanguard values its securities lending business despite passing on large fractions of securities lending revenue to its fundholders, as doing so allows the asset management company to offer a more competitive, lower-fee investment products to its clients. In contrast, when introducing a platform that facilitates competitive shares lending, there would be no continuation value that may provide commitment to secrecy. Having said that, new fintech developments such as smart contracts might facilitate the implementation of such trading arrangements without the need to rely on an intermediary platform operators that are subject to commitment problems. Such novel market structures could be an interesting avenue for future research.

7.2 Relation to Trading in Options Markets

In practice, agents wishing to bet against a security can do so not only by borrowing and selling the security but also by trading in options. For larger, more liquid stocks, options are often available and traded in centralized markets. One may wonder how this phenomenon relates to the main insights captured by our model.

If an options market were introduced in an environment like ours, hedge funds wishing to trade on negative information would use both venues (the options market and the securities lending market) in equilibrium, only if they were, at the margin, indifferent between these two venues. Indeed, Muravyev et al. (2022) find that empirically, options prices are consistent with the securities lending market and the options market being integrated in that borrowing costs are being passed on to buyers of options. A simple mechanism why such pricing may prevail in practice is the fact that options market makers face regulatory requirements to hedge their positions themselves by shorting the underlying securities and thereby incurring borrowing fees.

Since the buyer of a put option only needs to implement one trade (rather than a two-step transaction), it is the options market maker that not only faces these hedging costs but also adverse selection from informed traders. This adverse selection cost faced by market makers is aggravated by the fact that the trades realized in the centralized options markets do become public information immediately, implying that market makers cannot borrow and sell the security before the prices of the underlying security have already changed in response to this public signal. Both these costs are then reflected in options quotes and make the options market less advantageous for agents wishing to bet against an underlying security.

However, this is not to say that only one market would be used in equilibrium. Rather, a key equilibriating force determining how much order flow from prospective short sellers would be directed toward the options market versus toward the securities lending market is the relative quantity of liquidity motivated trades across markets. If liquidity-motivated traders approach primarily the centralized market for the underlying security (rather than the options market), then more of the order flow aiming to bet against a security would be optimally implemented through a standard two-step shorting transaction that leverages secrecy and the ability to directly trade against these liquidity traders.

In sum, the presence of an options market does not take away from the relevance of the features of the securities lending market we highlight. Fundamentally, this is because (1) options market makers also need to hedge positions (that is, they cannot take unbounded naked short positions), and (2) informed traders have incentives to exploit the presence of liquidity-motivated trades in the market of the underlying stock. Thus, the securities lending market is not redundant and short sellers benefit from it being set up in a way that avoids information leakages due to the two-step nature of short sales, as highlighted by our analysis.

8 Conclusion

In this paper, we examine the presence of market power in the securities lending market and evaluate its impact on different groups of investors and valuations. We document high market concentration, non-competitive fees, and low inventory utilization in the cross-section of stocks. Motivated by this evidence, we develop a tractable dynamic model to shed light on the conditions under which this current market structure benefits informed traders and shares lenders. While investors participating in shares lending have a clear preference for a delegated market structure, informed traders such as hedge funds share this preference when trading in illiquid securities, despite the fact that the fees they are charged are non-competitive. Key elements of our model are informed traders' concerns about information leakages and a recognition of the fact that shorting is a two-step transaction whereby securities are first borrowed and only thereafter can be sold.

Our model yields quantitative estimates of the impact of non-competitive fees on security lenders' stock valuations and the distribution of alphas that short sellers obtain from targeting different segments of the cross-section of stocks. Our results reveal that market power in securities lending markets can have material spill-over effects on valuations in stock markets, thereby distorting the informational content of stock prices. These findings are particularly relevant in the context of the literature on financial market feedback effects, which argues that information aggregated by stock prices is an important input to firms' investment decisions in practice (see Bond et al., 2012; van Binsbergen and Opp, 2019). Our results suggest that especially the prices of small stocks are substantially impacted by non-fundamental value components due to securities lending.

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A Order of Events in the Centralized Lending Market

In the setting with a centralized lending market, the logical order of events upon an innovation $dN_t = 1$ is generally identical to the one we laid out for the delegated market, with differences only pertaining to steps 2 and 4:

1. Dividend income realized at time t is collected by the agents who were the owners of the asset at time $t-$ (recall that t is an ex-dividend date).

2. Investors can post fees for lendable shares. Posted shares are tied up until settlement (step 7). Loan agreements start at time t and mature at the time of the next Poisson shock $dN_\tau = 1$ with $\tau > t$. At maturity, a borrower has to return the underlying asset and its dividend at that time to the lender.

3. A new hedge fund and a new cohort of liquidity investors arrive to the market.

4. The hedge fund can pick up fee quotes for lendable shares. The realized volume of borrowed shares becomes publicly observable.

5. Bid and ask quotes are posted by competitive investors in the centralized limit order market.

6. Investors trade shares at the posted bid and ask prices in the centralized limit order market.

7. Contracts are settled via delivery of the asset in accordance with all lending contracts and trades (steps 2 to 6 above).

B Proofs

B.1 Proof of Proposition 1

The buy-and-hold value $P(c)$ solves the Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned}0 =& -r_f P(c) + P'(c)c\mu_c + \frac{1}{2}P''(c)c^2\sigma_c^2 - P'(c)c\sigma_c\chi \\ &+ \lambda \cdot \left(\mathbb{E}\left[c \cdot e^v + P\left(c \cdot e^v\right) + F_v(v_\phi)\frac{\pi}{\rho}\phi\left(c \cdot e^v\right)\right] - P(c)\right). \end{aligned} \tag{31}$$

We conjecture and verify that $P(c)$ and the fee $\phi(c)$ are linear functions of c and we define $\tilde{P} \equiv \frac{P}{\lambda c}$ and $\tilde{\phi} \equiv \frac{\phi}{\lambda c}$. Plugging $P(c) = \tilde{P}\lambda c$ and $\phi(c) = \tilde{\phi}\lambda c$ into the HJB equation yields:

$$0 = -r_f\tilde{P}\lambda c + \tilde{P}\lambda c\mu_c - \tilde{P}\lambda c\sigma_c\chi + \lambda \cdot \left(c + \tilde{P}\lambda c + F_v(v_\phi)\frac{\pi}{\rho}\tilde{\phi}\lambda c e^v - \tilde{P}\lambda c\right), \tag{32}$$

where we use the fact that $\mathbb{E}[e^v] = 1$. We obtain:

$$\tilde{P} = \frac{1 + F_v(v_\phi)\frac{\pi}{\rho}\tilde{\phi}\lambda}{r_f + \sigma_c\chi - \mu_c}, \tag{33}$$

or equivalently:

$$P(c) = \frac{\lambda \cdot \left(c + F_v(v_\phi)\frac{\pi}{\rho}\phi(c)\right)}{r_f + \sigma_c\chi - \mu_c}. \tag{34}$$

Next, we compute the buy-and-hold value given the information set of an informed hedge fund, that is, conditional on observing the current value of v_t. Let $Q(c,v)$ denote this value which solves the HJB equation:

$$\begin{aligned}0 =& -r_f Q(c,v) + \frac{\partial Q(c,v)}{\partial c}c\mu_c + \frac{1}{2}\frac{\partial^2 Q(c,v)}{\partial c^2}c^2\sigma_c^2 - \frac{\partial Q(c,v)}{\partial c}c\sigma_c\chi \\ &+ \lambda \cdot \left(c \cdot e^v + P\left(c \cdot e^v\right) + F_v(v_\phi)\frac{\pi}{\rho}\phi\left(c \cdot e^v\right) - Q(c,v)\right). \end{aligned} \tag{35}$$

Conjecturing the solution $Q(v,c) = e^{v}\lambda c\tilde{P}$ and substituting it together with $P(c\cdot e^{v}) = e^{v}\lambda c\tilde{P}$ and $\phi\,(c\cdot e^{v}) = e^{v}\lambda c\tilde{\phi}$ into this HJB equation yields:

$$\begin{aligned} 0 = & -r_f e^{v}\lambda c\tilde{P} + e^{v}\lambda \tilde{P}c\mu_c - e^{v}\lambda\tilde{P}c\sigma_c\chi \\ & + \lambda\cdot\left(c\cdot e^{v} + e^{v}\lambda c\tilde{P} + F_v(v_\phi)\frac{\pi}{\rho}e^{v}\lambda c\tilde{\phi} - e^{v}\lambda c\tilde{P}\right), \end{aligned} \tag{36}$$

which is consistent with the earlier solution for $P(c)$ (see equation (34)). Thus, $Q(c,v) = P(c)e^{v}$.

B.2 Proof of Proposition 2

The liability value $L(v,c)$ solves the HJB equation:

$$0 = -\left(r_f + \sigma_c\chi - \mu_c\right)\cdot L(c,v) + \lambda\left(ce^{v} + A_1(c,v) - L(c,v)\right). \tag{37}$$

We conjecture that $L(v,c) = \lambda c\cdot\tilde{L}(v)$ and $A_1(c,v) = \lambda c\cdot e^{v}\tilde{A}_1$. Here we use the fact that the hedge fund observing v_{t-} knows that the scaled buy-and-hold value is $\tilde{Q}(v_{t-}) = \tilde{P}e^{v_{t-}}$ and similarly, the scaled ask price will be $\tilde{A}_1 e^{v_{t-}}$. Plugging in these conjectures, the HJB equation takes the form:

$$0 = -\left(r_f + \sigma_c\chi - \mu_c\right)\cdot\lambda c\cdot\tilde{L}(v) + \lambda\left(ce^{v} + \lambda c\cdot e^{v}\tilde{A}_1 - \lambda c\cdot\tilde{L}(v)\right). \tag{38}$$

which simplifies to:

$$\tilde{L}(v) = e^{v}\frac{1+\lambda\tilde{A}_1}{r_f + \sigma_c\chi - \mu_c + \lambda}, \tag{39}$$

or equivalently:

$$L(c,v) = e^{v}\frac{\lambda\cdot(c + A_1(c))}{r_f + \sigma_c\chi - \mu_c + \lambda}. \tag{40}$$

B.3 Proof of Proposition 3

The custodian knows that if it charges a loan fee $\phi(c) = \tilde{\phi}\lambda c$, all hedge fund types v satisfying

$$\tilde{\phi} \leq \tilde{B} - \tilde{L}(v_\phi) \tag{41}$$

will accept. Since borrowing hedge funds have to deliver both the dividend and the shares upon maturity, a shares lender does not lose any of the cash flows from the asset. The shares lender's scaled expected payoff from extra fee income when quoting a scaled loan fee $\tilde{\phi} > 0$ is then given by:

$$\begin{aligned} & \pi \Pr[B(c) - L(c, v_\phi) > \phi(c)] \cdot \phi(c) && (42) \\ =& \pi \Pr\left[B(c) - e^{v} \frac{\lambda(c + A_1(c))}{r_f + \sigma_c \chi - \mu_c + \lambda} > \phi(c)\right] \cdot \phi(c) \\ =& \pi \Pr\left[v < \log\left(\frac{(B(c) - \phi(c))(r_f + \sigma_c \chi - \mu_c + \lambda)}{\lambda(c + A_1(c))}\right)\right] \cdot \phi(c). && (43) \end{aligned}$$

Using the solutions, $A_1(c) = \lambda c \tilde{A}_1$, $B(c) = \lambda c \tilde{B}$, and $\phi(c) = \lambda c \tilde{\phi}$, we define the marginal hedge fund type v_ϕ (that is, the above-mentioned threshold value) choosing to borrow the asset:

$$v_\phi \equiv \log\left(\frac{(\tilde{B} - \tilde{\phi})(r_f + \sigma_c \chi - \mu_c + \lambda)}{1 + \lambda \tilde{A}_1}\right), \tag{44}$$

so that:

$$\tilde{\phi} = \tilde{B} - e^{v_\phi} \frac{1 + \lambda \tilde{A}_1}{r_f + \sigma_c \chi - \mu_c + \lambda}. \tag{45}$$

Using this definition, we can express the shares lender's profit as a function of the marginal hedge fund type v_ϕ:

$$\tilde{\Pi}(v_\phi) = \pi \cdot F_v(v_\phi) \cdot \left(\tilde{B} - e^{v_\phi} \frac{1 + \lambda \tilde{A}_1}{r_f + \sigma_c \chi - \mu_c + \lambda}\right). \tag{46}$$

The shares lender's marginal profit of increasing v_ϕ is

$$\tilde{\Pi}'(v_\phi) = \pi \cdot f_v(v_\phi)\left(\tilde{B} - \frac{1+\lambda \tilde{A}_1}{r_f+\sigma_c\chi-\mu_c+\lambda}e^{v_\phi}\right) \\ - \pi \cdot F_v(v_\phi) \cdot \frac{1+\lambda \tilde{A}_1}{r_f+\sigma_c\chi-\mu_c+\lambda}e^{v_\phi}. \tag{47}$$

At the optimum, the marginal profit is equal to zero, that is, v_ϕ solves $\Pi'(v_\phi) = 0$, or equivalently:

$$\frac{f_v(v_\phi)}{F_v(v_\phi)}\left(\frac{\tilde{B}\cdot(r_f+\sigma_c\chi-\mu_c+\lambda)}{1+\lambda\tilde{A}_1}e^{-v_\phi}-1\right) = 1, \tag{48}$$

which can be rewritten as:

$$\frac{f_v(v_\phi)}{F_v(v_\phi)} \cdot \left(\frac{B(c)}{L(c,v_\phi)}-1\right) = 1. \tag{49}$$

B.4 Generalized Borrowing Motives and Proof of Proposition 4

In this appendix section, we generalize our baseline model to incorporate private-value shocks to hedge funds, and characterize in this more general environment the equilibria for both the delegated and centralized market structures (this generalized setting nests the baseline model).

In practice, some institutions participating in the shares lending market may do so for non-informational reasons. For example, a financial institution may wish to offset an existing exposure for regulatory or risk management reasons. To capture this feature, we extend our baseline model in this section by introducing private-value shocks affecting hedge funds.

Suppose hedge funds not only obtain information upon entry but also are subject to private-value shocks that imply that they apply an additional discount factor e^b to the future cash flows associated with the asset, where $b \sim Normal(-\frac{\sigma_b^2}{2}, \sigma_b)$. When $b > 0$ hedge funds assign a private-value premium, otherwise a discount (for $b < 0$) to the cash flows. Apart from this private-value component, hedge funds still also observe the common value component v. A hedge fund's decision-relevant type is now given by $w \equiv v + b$, where $w \sim Normal(-\frac{\sigma_v^2+\sigma_b^2}{2}, \sqrt{\sigma_v^2+\sigma_b^2})$. Similar to before, there is a threshold type, which we denote by w_ϕ, that corresponds to the hedge fund type that is just indifferent between borrowing and not borrowing at the posted fee ϕ.

When buying the asset upon entry, a hedge fund now assigns the value

$$Q(c,w) = e^w \cdot P(c), \tag{50}$$

where $P(c)$ mimics our earlier solution in the baseline model (compare equation (5)):

$$P(c) = \frac{\lambda \cdot (c + F_w(w_\phi)\frac{\pi}{\rho}\phi(c))}{r_f + \sigma_c\chi - \mu_c}. \tag{51}$$

Analogously, when taking a short position upon entry, a hedge fund now assigns the following value to the cash flows associated with its liability:

$$L(c,w) = e^w \frac{\lambda \cdot (c + A_1(c))}{r_f + \sigma_c\chi - \mu_c + \lambda}. \tag{52}$$

which again mimics the solution in our baseline model (compare equation (15)). A hedge fund's maximum willingness to pay is then given by $(B(c) - L(w,c))$.

Delegated securities lending market. In the setting with private-value shocks, the equations determining the bid and ask prices conditional on a delegated market structure are given by:

$$\frac{B}{P} = \frac{\pi F_w(w_\phi)\,\mathbb{E}[e^v|w \le w_\phi] + (1-\pi)}{\pi F_w(w_\phi) + (1-\pi)}, \tag{53}$$

$$\frac{A_0}{P} = \frac{\pi(1 - F_w(\log\frac{A_0}{P}))\,\mathbb{E}[e^v|w \ge \log\frac{A_0}{P}] + (1-\pi) + F_w(w_\phi)\frac{\pi}{\rho}\frac{\phi}{P}}{\pi(1 - F_w(\log\frac{A_0}{P})) + 1 - \pi}, \tag{54}$$

$$\frac{A_1}{P} = \frac{\pi(1 - F_w(\log\frac{A_1}{P}))\,\mathbb{E}[e^v|w \ge \log\frac{A_1}{P}] + \pi + (1-\pi) + (1+\pi)F_w(w_\phi)\frac{\pi}{\rho}\frac{\phi}{P}}{\pi(1 - F_w(\log\frac{A_1}{P})) + \pi + (1-\pi)}, \tag{55}$$

where we can rewrite the conditional expectation as follows:

$$\mathbb{E}[e^v|w < w_\phi] = \mathbb{E}[\mathbb{E}[e^v|v < w_\phi - b]] = \int_{-\infty}^{+\infty} f_b(b)\frac{F_v(w_\phi - b)}{F_w(w_\phi)}\,\mathbb{E}[e^v|v < w_\phi - b]db. \tag{56}$$

Since

$$\mathbb{E}[e^{v_t}|v_t \leq w_\phi - b] = \frac{F_v(w_\phi - b - \sigma_v^2)}{F_v(w_\phi - b)}, \tag{57}$$

we obtain:

$$\mathbb{E}[e^v|w < w_\phi] = \int_{-\infty}^{+\infty} f_b(b)\frac{F_v(w_\phi - b - \sigma_v^2)}{F_w(w_\phi)}db. \tag{58}$$

Similarly, we can compute the conditional expectation:

$$\mathbb{E}[e^v|w \geq w_\phi] = \int_{-\infty}^{+\infty}\int_{w_\phi - b}^{+\infty} \frac{f_b(b)f_v(v)}{1 - F_w(w_\phi)}e^v dvdb \tag{59}$$

$$= \int_{-\infty}^{+\infty} f_b(b)\frac{1 - F_v(w_\phi - b)}{1 - F_w(w_\phi)}\mathbb{E}[e^v|v \geq w_\phi - b]db. \tag{60}$$

Since $\mathbb{E}[e^v|v \geq w_\phi - b] = \frac{F_v(-w_\phi + b)}{1 - F_v(w_\phi - b)}$, we obtain:

$$\mathbb{E}[e^v|w \geq w_\phi] = \int_{-\infty}^{+\infty} f_b(b)\frac{F_v(-w_\phi + b)}{1 - F_w(w_\phi)}db. \tag{61}$$

Analogously to the baseline model, the delegated shares lender chooses the optimal threshold type w_ϕ, which pins dow the loan fee

$$\phi(c) = B(c) - L(c, w_\phi). \tag{62}$$

The shares lender maximizes

$$\tilde{\Pi}(w_\phi) = \pi \cdot F_w(w_\phi) \cdot \left(\tilde{B} - e^{w_\phi}\frac{1 + \lambda\tilde{A}_1}{r_f + \sigma_c\chi - \mu_c + \lambda}\right). \tag{63}$$

The shares lender's marginal profit of increasing w_ϕ is

$$\tilde{\Pi}'(w_\phi) = \pi \cdot f_w(w_\phi)\left(\tilde{B} - \frac{1 + \lambda\tilde{A}_1}{r_f + \sigma_c\chi - \mu_c + \lambda}e^{w_\phi}\right)$$
$$- \pi \cdot F_w(w_\phi) \cdot \frac{1 + \lambda\tilde{A}_1}{r_f + \sigma_c\chi - \mu_c + \lambda}e^{w_\phi}. \tag{64}$$

At the optimum, the marginal profit is equal to zero, that is, w_ϕ solves $\Pi'(w_\phi) = 0$, or equivalently:

$$\frac{f_w(w_\phi)}{F_w(w_\phi)}\left(\frac{\tilde{B}\cdot(r_f+\sigma_c\chi-\mu_c+\lambda)}{1+\lambda\tilde{A}_1}e^{-w_\phi}-1\right)=1, \tag{65}$$

which can be rewritten as:

$$\frac{f_w(w_\phi)}{F_w(w_\phi)}\cdot\left(\frac{B(c)}{L(c,w_\phi)}-1\right)=1. \tag{66}$$

Competitive securities lending market. In a competitive securities lending market, the lending fee is zero. Thus, even a hedge fund who does not want to short can nonetheless costlessly borrow the shares (at a fee equal to zero), hold them for the length of the contract and then redeliver them to the lender. This action can act as a signal and thus, we need to condition any quotes on whether there is new borrowing demand. Conditional on having borrowed, a hedge fund can take three actions: (1) sell the security at the bid, (2) keep the security and also buy more of the security at the prevailing ask price, (3) keep the security and neither buy nor sell the security. Option (3) is strictly dominated by options (1) and (2) conditional on the hedge fund actually wishing to trade at either the bid or ask price. Moreover, conditional on not wishing to trade, the hedge fund could still borrow and take option 3 (not trade). We assume the tie-breaker rule that such a hedge fund does not borrow in this case (which is supported by an arbitrarily small exogenous cost of taking the time to borrow).

As in the baseline analysis, we also need to distinguish the cases where there is preexisting short interest and no preexisting short interest. We use the first entry of the subscript $(\cdot|\cdot)$ to indicate whether there is preexisting short interest, and the second subscript to represent whether there is new borrowing. Note that after trading volume has been realized, it can be inferred from public information whether a hedge fund that had borrowed shares did not sell them at the bid price (but rather bought at the ask price). In that case, the preexisting short interest is 0 and agents know that there is no hedge fund in the market that will need to repurchase the asset at the end of the contract period.

Let $w_{\phi_{0|1}}$ and $w_{\phi_{1|1}}$ denote the threshold types for hedge funds wishing to short sell the asset (that is, borrow and sell at the bid). Conditional having a type below the threshold type, a hedge fund borrows the asset with probability 1 and immediately sells it at the bid price B with probability 1. The marginal type is the hedge fund that just breaks even at a zero fee when *borrowing and selling*, that is, $w_{\phi_{0|1}}$ and $w_{\phi_{1|1}}$ solve:

$$B_{0|1}(c) - L(c, w_{\phi_{0|1}}) = 0, \tag{67}$$

$$B_{1|1}(c) - L(c, w_{\phi_{1|1}}) = 0. \tag{68}$$

Bid prices. Consider liquidity providers' beliefs regarding the probability that, conditional on observing new borrowing, a hedge fund will pick up the ask quote. Let $\nu_{0|1}$ and $\nu_{1|1}$ denote the probabilities that hedge funds *buy* at the ask price after having borrowed, conditional on no preexisting short interest and existing short interest, respectively. Since option (3) is not used in equilibrium, the conditional probability that a hedge fund *sells* at the bid after having borrowed is then given by $(1-\nu_{0|1})$ and $(1-\nu_{1|1})$, respectively.

Conditional on new borrowing having occurred, the bid prices then solve (conditional on no preexisting short interest and conditional on preexisting short interest, respectively):

$$\frac{B_{0|1}}{P} = \frac{(1-\nu_{0|1})\pi\,\mathbb{E}[e^{v}|w \leq w_{\phi_{0|1}}] + (1-\pi)}{(1-\nu_{0|1})\pi + 1 - \pi}, \tag{69}$$

$$\frac{B_{1|1}}{P} = \frac{(1-\nu_{1|1})\pi\,\mathbb{E}[e^{v}|w \leq w_{\phi_{1|1}}] + (1-\pi)}{(1-\nu_{1|1})\pi + 1 - \pi}. \tag{70}$$

In contrast, conditional on no new borrowing having occurred, the bid is priced to reflect that only liquidity traders might pick up the bid. That is, there is no adverse selection adjustment:

$$\frac{B_{0|0}}{P} = \frac{B_{1|0}}{P} = 1. \tag{71}$$

Ask prices. Again, we need to distinguish cases by whether there is preexisting short interest and by whether new borrowing has occurred.

First, consider the case where there is no preexisting short interest. Let $A_{0|0}$ denote the ask price conditional on the state where neither old nor new borrowing demand exists. Further let $A_{0|1}$ denote the ask price conditional no old borrowing demand but new borrowing demand. The conditional ask prices satisfy:

$$\frac{A_{0|0}}{P} = \frac{v_{0|0} \cdot \pi \cdot \mathbb{E}[e^v | w \geq \log \frac{A_{0|0}}{P}] + (1-\pi)}{v_{0|0}\pi + (1-\pi)}, \tag{72}$$

$$\frac{A_{0|1}}{P} = \frac{v_{0|1} \cdot \pi \cdot \mathbb{E}[e^v | w \geq \log \frac{A_{0|1}}{P}] + (1-\pi)}{v_{0|1}\pi + (1-\pi)}. \tag{73}$$

In principle, either conditional ask price could be advantageous for a hedge fund, depending on liquidity providers beliefs about the likelihood with which a hedge fund with positive news borrows versus does not borrow. A pure-strategy equilibrium is not sustainable since conditional on the market believing that the hedge fund always chooses one option (say a purchasing hedge fund is believed to never simultaneously borrow), the ask price conditional on borrowing occurring would reflect the belief that only uninformed traders pick up that ask quote. Yet conditional on this belief, the ask quote would feature no adverse selection adjustment, causing the hedge fund to switch to borrowing securities while simultaneously buying them. In equilibrium, the ask prices $A_{0|0}$ and $A_{0|1}$ therefore must be identical, such that a hedge fund intending to buy at the ask price does not have a strict preference over whether to borrow or not to borrow. The condition that $A_{0|0} = A_{0|1}$ implies that

$$v_{0|0} = v_{0|1}. \tag{74}$$

We define $A_0 \equiv A_{0|0} = A_{0|1}$ and $v_0 \equiv v_{0|0} = v_{0|1}$. We will characterize v_0 below.

Second, consider the case where there is preexisting short interest. The conditional ask prices then account for the additional uninformed demand from hedge funds having to redeliver π units of shares. This yields the relations:

$$\frac{A_{1|0}}{P} = \frac{\pi + v_{1|0} \cdot \pi \cdot \mathbb{E}[e^v | w \geq \log \frac{A_{1|0}}{P}] + (1-\pi)}{\pi + v_{1|0}\pi + (1-\pi)}, \tag{75}$$

$$\frac{A_{1|1}}{P} = \frac{\pi + v_{1|1} \cdot \pi \cdot \mathbb{E}[e^v | w \geq \log \frac{A_{1|1}}{P}] + (1-\pi)}{\pi + v_{1|1}\pi + (1-\pi)}. \tag{76}$$

Since it is costless to borrow, the ask prices $A_{1|0}$ and $A_{1|1}$ must also be identical in equilibrium, such that a hedge fund intending to buy at the ask price does not have a strict preference over whether to borrow or not to borrow. The condition that $A_{1|0} = A_{1|1}$ implies that

$$\nu_{1|0} = \nu_{1|1}. \tag{77}$$

We define $A_1 \equiv A_{1|0} = A_{1|1}$ and $\nu_1 \equiv \nu_{1|0} = \nu_{1|1}$.

Next, we solve for ν_0 and ν_1. Conditional on there being preexisting short interest, the probability that a hedge fund buys the asset at the ask price is given by:

$$\begin{aligned} &\Pr[\text{new borrowing}|\text{existing borrowing}] \cdot \nu_{1|1} \\ &+(1 - \Pr[\text{new borrowing}|\text{existing borrowing}]) \cdot \nu_{1|0} \\ &=1 - F_w(w \geq \log \frac{A_1}{P}). \end{aligned} \tag{78}$$

Using $\nu_1 \equiv \nu_{1|0} = \nu_{1|1}$ yields:

$$\nu_1 = 1 - F_w(w \geq \log \frac{A_1}{P}). \tag{79}$$

Analogously, we obtain:

$$\nu_0 = 1 - F_w(w \geq \log \frac{A_0}{P}). \tag{80}$$

Finally, we determine the probability of borrowing, which occurs either by hedge funds wishing to short or by hedge funds wishing to buy the asset. Conditional on preexisting short interest we obtain the relation:

$$\begin{aligned} &\Pr[\text{new borrowing}|\text{existing borrowing}] \\ &= F_w(w_{\phi_{1|1}}) + \Pr[\text{new borrowing}|\text{existing borrowing}] \cdot \nu_{1|1}. \end{aligned} \tag{81}$$

Which yields

$$\Pr[\text{new borrowing}|\text{existing borrowing}] = \frac{F_w(w_{\phi_{1|1}})}{(1 - \nu_{1|1})} = \frac{F_w(w_{\phi_{1|1}})}{F_w(w \geq \log \frac{A_1}{P})}. \tag{82}$$

Analogously, we obtain for the case without preexisting borrowing:

$$\text{Pr[new borrowing|no existing borrowing]} = \frac{F_w(w_{\phi_{0|1}})}{(1-\nu_{0|1})} = \frac{F_w(w_{\phi_{0|1}})}{F_w(w \geq \log \frac{A_0}{P})}. \tag{83}$$

B.5 Proof of Proposition 5

The HJB equation corresponding to (24), scaled by λc, is given by:

$$0 = -(r_f + \sigma_c(s)\chi - \mu_c(s)) \cdot \tilde{P}(s) + (1 + \psi(s)) + (\Lambda(s) \odot \mathbf{U}(s)) \cdot \tilde{\mathbf{P}} \quad \forall s, \tag{84}$$

where $\odot$ denotes the Hadamard product and where the fee-to-dividend ratio is given by:

$$\psi(s) = \frac{\lambda F_{v,s}(v_\phi(s)) \frac{\pi}{\rho(s)} \phi(s,c)}{\lambda c} = \lambda F_{v,s}(v_\phi(s)) \frac{\pi}{\rho(s)} \tilde{\phi}(s) \tag{85}$$

since $\phi(s,c)$ again scales linearly with c. Moreover, the HJB equation associated with the present value without fee income (25) is given by:

$$0 = -(r_f + \sigma_c(s)\chi - \mu_c(s)) \cdot \tilde{P}^e(s) + 1 + (\Lambda(s) \odot \mathbf{U}(s)) \cdot \tilde{\mathbf{P}}^e \quad \forall s. \tag{86}$$

These sets of linear equations yield closed-form solutions for the vectors of price-dividend ratios:

$$\tilde{\mathbf{P}} = (\Lambda \odot \mathbf{U} - diag(r_f + \sigma_c(s)\chi - \mu_c(s)))^{-1}(\mathbf{1} + \psi), \tag{87}$$

$$\tilde{\mathbf{P}}^e = (\Lambda \odot \mathbf{U} - diag(r_f + \sigma_c(s)\chi - \mu_c(s)))^{-1}\mathbf{1}. \tag{88}$$

Analogously to our baseline model, we also obtain $Q(c,v,s) = \lambda c \tilde{P}(s) e^v$. Similarly, we again conjecture that $L(v,c,s) = \lambda c \cdot \tilde{L}(v,s)$ and $A_1(c,v,s) = \lambda c \cdot e^v \tilde{A}_1(s)$. A hedge fund observing v_{t-} knows that the scaled buy-and-hold value is $\tilde{Q}(v_{t-},s) = \tilde{P}(s)e^{v_{t-}}$ and similarly, the scaled ask price will be $\tilde{A}_1(s)e^{v_{t-}}$. Using these relations, the HJB equation for the liability $\tilde{L}(v,s)$ incurred from borrowing shares takes the form:

$$0 = -(r_f + \sigma_c(s)\chi - \mu_c(s)) \cdot \lambda c \cdot \tilde{L}(v,s) + \lambda(ce^v + \lambda c \cdot e^v \tilde{A}_1(s) - \lambda c \cdot \tilde{L}(v,s)) \\ + \lambda c \cdot (\Lambda(s) \odot \mathbf{U}(s)) \cdot \tilde{\mathbf{L}}(v), \quad (89)$$

which simplifies to:

$$\tilde{\mathbf{L}}(v) = e^v \cdot (\Lambda \odot \mathbf{U} - diag(r_f + \sigma_c(s)\chi - \mu_c(s)) + \lambda)^{-1}(\mathbf{1} + \lambda \tilde{\mathbf{A}}_1). \quad (90)$$

This solution implies that $\tilde{L}(v_\phi, s) = e^{v_\phi}\,\mathbb{E}[\tilde{L}(v,s)|s]$. Expressing the shares lender's profit as a function of the marginal hedge fund type $v_\phi(s)$ we obtain:

$$\tilde{\Pi}(v_\phi(s)) = \pi \cdot F_{v,s}(v_\phi(s)) \cdot \left(\tilde{B}(s) - e^{v_\phi(s)}\,\mathbb{E}[\tilde{L}(v,s)|s]\right). \quad (91)$$

The shares lender's marginal profit of increasing v_ϕ is

$$\tilde{\Pi}'(v_\phi(s)) = \pi \cdot f_{v,s}(v_\phi(s))\left(\tilde{B}(s) - \mathbb{E}[\tilde{L}(v,s)|s]e^{v_\phi(s)}\right) - \pi \cdot F_{v,s}(v_\phi(s)) \cdot \mathbb{E}[\tilde{L}(v,s)|s]e^{v_\phi(s)}. \quad (92)$$

At the optimum, the marginal profit is equal to zero, that is, $v_\phi(s)$ solves $\Pi'(v_\phi(s)) = 0$, or equivalently:

$$\frac{f_{v,s}(v_\phi(s))}{F_{v,s}(v_\phi(s))}\left(\frac{\tilde{B}(s)}{e^{v_\phi(s)}\,\mathbb{E}[\tilde{L}(v,s)|s]} - 1\right) = 1. \quad (93)$$

C Data Appendix

Our sample of U.S. stocks is constructed by combining CRSP data with the FSTE Russell index membership data, as well as securities lending data from Markit. The sample period is from 2007 through 2021, during which Markit has a good coverage of CRSP stocks.

Information about prices, returns, and dividends are from CRSP. For dividend payments, we consider ordinary cash dividends paid in US dollars (CRSP distribution codes starting with 12). Because we are interested in all cash dividends that investors can earn by holding shares, we include dividend payments at any frequency.[23] As our analysis is

[23] 91.1% of all dividend observations are quarterly, 1.3% of dividends are semi-annual, 0.2% are annual, and the rest are of other, unknown, or missing frequency.

at the quarterly frequency, we calculate the quarterly dividend of a stock as the sum of all dividends distributed in a quarter.

Our securities lending data are from Markit Securities Finance Data Analytics (formerly Data Explorers). Markit provides a variety of lending indicators, from which we select data fields that are necessary for calculating lending fee income and fee yield earned by shares lenders: shares lending supply, shares borrowing demand, and borrowing costs. Specifically, we use Beneficial Owner's Inventory Value in Markit to measure the dollar amount of shares lending supply, and Value on Loan to measure the dollar amount of shares borrowing demand. Markit provides three important fields on borrowing costs: Daily Cost of Borrow Score (DCBS), Indicative Fee, and Simple Average Fee. The DCBS is a 1–10 categorization that describes how expensive a stock is to borrow, with 1 being the cheapest and 10 being the most expensive. Markit computes DCBS based on the proprietary data of actual lending fee quotes which are received from securities dealers but are not allowed to re-distribute. Indicative Fee is a derived rate using the Markit's proprietary analytics and dataset of both contributed borrowing costs between Agent Lenders and Prime Brokers as well as contributed rates from hedge fund participants. However, Indicative Fee can not serve the our purpose of computing lending fee income because it is not the *actual* rate Prime Brokers quote or charge but rather the *expected* rate for investors such as hedge fund to borrow shares in securities lending markets. Simple Average Fee (we will call it Fee later) is the equal-weighted average of actual fees across all outstanding loan contracts on a stock. Although Fee is suitable for calculating lending fee income, its data histories are incomplete in Markit. Actually, it is not unusual to have a missing Fee for a stock-day with positive shares on loan, especially during the early sample period.

We propose a method of filling missing Fee based on three important data features: 1) Fee observations of stocks with the same DCBS tend to have very similar magnitudes; 2) Fee and Indicative Fee are highly correlated with each other (the correlation is about 0.8); 3) both DCBS and Indicative Fee are well populated in Markit data. These features suggest that we can fill a missing Fee of a stock on a given day with the average of non-missing Fees of stocks with similar Indicative Fees and in the same DCBS category. Specifically, we implement our method in three steps: 1) sorting stocks with the same DCBS on a given day into 10 bins by their Indicative Fees; 2) taking the average of non-missing Fees for each combination of DCBS and Indicative Fee bin; 3) filling a missing Fee with the average Fee of stocks in the matched combination. To check the validity of our method, we simulate the "filled" Fees for non-missing Fee observations and we find the correlation between the "filled" Fees and actual Fees above 0.9 for all DCBS categories.

With lending fees data, we calculate one of our key variables, lending fee income, as value on loan times lending fees on a day. Similar to the quarterly dividend, the quarterly fee income is the sum of daily fee incomes over a quarter.

For some of our empirical market concentration measures it is useful to make adjustments for the coverage that the Markit Database has of overall securities lending market activity. Markit reports that its securities lending data is collected throughout the day from more than 85% of global securities finance practitioners, including custodial banks, agent lenders, sell-side brokers, asset managers and hedge funds (see Markit (2012)). Correspondingly, when making adjustments for Markit coverage, we use the reported 85% (as in Muravyev et al., 2022). Moreover, we confirm that Markit covers the vast majority of US stocks; the percentage of the number of stocks covered by Markit increases from 85% to 98% over the sample period from 2007 to 2021.[24]

C.1 Sorting Procedure

The Russell Indexes have become the leading US equity benchmarks and have been widely accepted by institutional investors for their academic integrity and investor usability. One key feature of the Russell indexes is that they are reconstituted purely based on the rank of market capitalization of stocks. Russell Top 200, Russell Mid-Cap, Russell 1000, Russell 2000, and Russell Micro-Cap are commonly used indexes among practitioners. In total, the Russell universe includes 4000 stocks. In our empirical analysis, we split the Russell universe into four mutually exclusive size categories (Large-Cap, Mid-Cap, Small-Cap, and Micro-Cap) as described below. The remaining US stocks are too small to be included in the Russell universe so we put them into an additional category (Nano-Cap). To be consistent with the Russell Index constituents, we consider shares listed on all US exchanges, including not only common shares of US companies (CRSP share code 10 or 11), but also shares of companies incorporated outside the United States (share code 12) and REITs (share code 18).

We form 25 size and fee yield groups by the following conditional sorting procedure. At the beginning of each quarter, we first classify stocks into the 5 size categories as follows

- Large-Cap: stocks in Russell Top 200, which consists of the largest 200 members in Russell 1000

[24] We merge CRSP and Markit using historical CUSIPs, which are available in both databases.

- Mid-Cap: stocks in Russell Mid-Cap, which consists of the smallest 800 members in Russell 1000
- Small-Cap: the largest 1000 members in Russell 2000
- Micro-Cap: stocks in Russell Micro-Cap, which consists of the smallest 1000 members in Russell 2000 plus the largest 1000 stocks outside Russell 2000
- Nano-Cap: all remaining stocks

For each of the above size groups, we further sort stocks into five bins by the cutoffs of fee yield percentiles, 80%, 90%, 95%, and 98%. We select these cutoffs because of the strong right-skewness of fee yields in the cross-section.

D Tables

Panel A. Market Share of the Lender with the Highest Value on Loan

Fee Yield Percentiles	Large-Cap (1)	Mid-Cap (2)	Small-Cap (3)	Micro-Cap (4)	Nano-Cap (5)
$\leq$ 80%	37.09	37.82	39.73	40.78	64.06
80-90%	31.41	31.48	34.20	38.92	56.53
90-95%	30.01	30.26	31.70	37.38	55.79
95-98%	28.83	28.49	31.05	37.09	54.86
$\geq$ 98%	27.93	28.26	31.34	38.56	57.13

Panel B. Market Share of the Lender with the Second Highest Value on Loan

Fee Yield Percentiles	Large-Cap (1)	Mid-Cap (2)	Small-Cap (3)	Micro-Cap (4)	Nano-Cap (5)
$\leq$ 80%	19.70	21.91	23.40	22.59	21.72
80-90%	19.13	21.49	21.75	21.08	22.94
90-95%	19.64	20.83	20.52	20.42	22.45
95-98%	19.93	19.57	19.57	20.68	22.93
$\geq$ 98%	20.24	18.02	20.26	20.70	22.64

Table 1: Concentration of Lenders' Value on Loan (Markit Data)
This table shows the concentration of lenders' value on loan for the 25 size and fee yield groups. For each stock we obtain the market share of value on loan of the top 2 lenders from Markit Securities Lending Database. We calculate the group-level market share of the top 2 lenders as

$$MarketShare_t^i = \frac{\sum_j ValueOnLoan_{j,t}^i \times MarketShare_{i,t}^i}{\sum_j ValueOnLoan_{j,t}^i}, \quad i = 1, 2, \dots 25,$$

where $MarketShare_{j,t}^i$ is market share of the (second) largest lender and $ValueOnLoan_{j,t}^i$ is value of shares on loan for stock j in group i. Panel A reports the time-series average of market shares of the lender with the highest value on loan. Panel B reports the time-series average of market shares of the lender with the second highest value on loan. Market shares are reported in percent. The sample period is from 2007 through 2021.

Panel A. Market Share of the Lender with the Highest Value on Loan

Fee Yield Percentiles	Large-Cap (1)	Mid-Cap (2)	Small-Cap (3)	Micro-Cap (4)	Nano-Cap (5)
$\leq$ 80%	30.14	30.51	32.79	33.21	51.42
80-90%	26.56	26.78	29.06	32.79	47.77
90-95%	25.46	25.83	26.90	31.07	46.51
95-98%	24.53	24.13	26.46	30.75	45.42
$\geq$ 98%	23.44	23.48	26.79	32.75	47.58

Panel B. Market Share of the Lender with the Second Highest Value on Loan

Fee Yield Percentiles	Large-Cap (1)	Mid-Cap (2)	Small-Cap (3)	Micro-Cap (4)	Nano-Cap (5)
$\leq$ 80%	16.51	18.50	19.82	19.04	18.72
80-90%	16.18	18.24	18.45	18.09	19.28
90-95%	16.75	17.68	17.39	17.44	19.74
95-98%	16.82	16.63	16.51	17.73	19.82
$\geq$ 98%	16.18	15.61	17.49	17.87	19.77

Table 2: Concentration of Lenders' Value on Loan (Markit-coverage Adjusted)
The table reports the concentration of lenders' value on loan for the 25 size and fee yield groups, using a conservative measure that accounts for coverage of the Markit database. In a first step, for each stock, we obtain the top 2 lenders' market shares for value on loan as reported by the Markit Securities Lending Database. In a second step, we calculate the group-level market share of the top 2 lenders as the fee income-weighted average of market shares across stocks in the group,

$$MarketShare_t^i = \frac{\sum_j LendingFeeIncome_{j,t}^i \times MarketShare_{j,t}^i}{\sum_j LendingFeeIncome_{j,t}^i}, \quad i = 1, 2, \ldots 25,$$

where $LendingFeeIncome_{j,t}^i$ is the sum of daily lending fee income of stock j in quarter t, and $MarketShare_{j,t}^i$ is the fee income-weighted average of daily market shares, multiplied by the Markit coverage of 85% (see Appendix C), for stock j in quarter t,

$$MarketShare_{j,t}^i = \frac{\sum_d MarketShare_{j,t,d}^i \times MarkitCoverage \times LendingFeeIncome_{j,t,d}^i}{\sum_d LendingFeeIncome_{j,t,d}^i}.$$

This measure is conservative in the sense that it assumes that 100% of the security lending activity not covered by Markit involves institutions that are distinct from the top 2 institutions identified by Markit.

Lending Fee (annual, in percent)

Fee Yield Percentiles	Large-Cap (1)	Mid-Cap (2)	Small-Cap (3)	Micro-Cap (4)	Nano-Cap (5)
$\leq$ 80%	0.285	0.276	0.290	0.498	3.887
80-90%	0.287	0.305	0.487	2.188	18.223
90-95%	0.288	0.463	1.205	6.254	31.631
95-98%	0.313	1.263	3.150	14.340	48.157
$\geq$ 98%	2.204	6.880	12.242	38.425	75.141

Table 3: Lending Fee

This table reports the time-series average of lending fees for the 25 size and fee yield groups. The lending fee of group i in quarter t is computed as

$$LendingFee_t^i = \frac{\sum_j ValueOnLoan_{j,t}^i \times LendingFee_{i,t}^i}{\sum_j ValueOnLoan_{j,t}^i}, \quad i = 1, 2, \ldots 25,$$

where $LendingFee_{j,t}^i$ and $ValueOnLoan_{j,t}^i$ are lending fees and value of shares on loan for stock j in group i, respectively. Lending fees are annualized and reported in percent. The sample period is from 2007 through 2021.

Panel A. Utilization of Inventory (in percent)

Fee Yield Percentiles	Large-Cap (1)	Mid-Cap (2)	Small-Cap (3)	Micro-Cap (4)	Nano-Cap (5)
$\leq 80\%$	1.10	3.85	9.70	6.64	2.50
80-90%	6.69	25.94	36.90	46.29	46.78
90-95%	11.97	36.72	47.80	62.66	62.10
95-98%	20.09	46.87	58.62	72.21	71.85
$\geq 98\%$	35.32	62.71	70.65	78.73	81.94

Panel B. Alternative Utilization of Inventory (in percent)

Fee Yield Percentiles	Large-Cap (1)	Mid-Cap (2)	Small-Cap (3)	Micro-Cap (4)	Nano-Cap (5)
$\leq 80\%$	2.39	9.86	17.96	20.83	30.96
80-90%	8.12	27.41	39.30	52.92	54.20
90-95%	13.30	38.91	52.16	67.92	66.33
95-98%	21.90	51.41	64.55	76.28	74.86
$\geq 98\%$	52.30	74.40	78.88	82.28	84.84

Table 4: Utilization of Inventory

This table reports the time-series average of utilizations of inventory for the 25 size and fee yield groups. In Panel A, the utilization of a group is computed as the aggregate value on loan divided by the aggregate value of shares in inventory across stocks in the group,

$$Utilization_t^i = \frac{\sum_j ValueOnLoan_{j,t}^i}{\sum_j ValueInventory_{j,t}^i}, \quad i = 1, 2, \ldots 25.$$

In Panel B, the (alternative) utilization of a group is computed as the fee income-weighted average of utilizations across stocks in the group,

$$AlternativeUtilization_t^i = \frac{\sum_j Utilization_{j,t}^i \times LendingFeeIncome_{j,t}^i}{\sum_j LendingFeeIncome_{j,t}^i}, \quad i = 1, 2, \ldots 25,$$

where $LendingFeeIncome_{j,t}^i$ is the sum of daily lending fee incomes of stock j in quarter t, and $Utilization_{j,t}^i$ is the fee income-weighted average of daily utilizations of stock j in quarter t,

$$Utilization_{j,t}^i = \frac{\sum_d Utilization_{j,t,d}^i \times LendingFeeIncome_{j,t,d}^i}{\sum_d LendingFeeIncome_{j,t,d}^i}.$$

Utilization are in percentage. The sample period is from 2007 through 2021.

Fee Yield (annual, in percent)

Fee Yield Percentiles	Large-Cap (1)	Mid-Cap (2)	Small-Cap (3)	Micro-Cap (4)	Nano-Cap (5)
$\leq$ 80%	0.003	0.011	0.0295	0.028	0.100
80-90%	0.019	0.081	0.193	0.970	10.815
90-95%	0.035	0.184	0.616	3.873	22.803
95-98%	0.065	0.668	1.936	10.090	37.242
$\geq$ 98%	0.850	4.446	8.675	29.315	63.955

Table 5: Fee Yield

This table reports the time-series average of fee yields for the 25 size and fee yield groups. The fee yield of group i in quarter t is computed as

$$FeeYield_t^i = \frac{\sum_j LendingFeeIncome_{j,t}^i}{\sum_j InventoryValue_{j,t}^i}, \quad i = 1, 2, \ldots 25,$$

where $LendingFeeIncome_{j,t}^i$ is the amount of fee incomes collected by lenders from their shares on loan in quarter t and $InventoryValue_{j,t}^i$ is the value of lenders' shares in inventory. Fee yields are annualized and reported in percent. The sample period is from 2007 through 2021.

Dividend Yield (annual, in percent)

Fee Yield Percentiles	Large-Cap (1)		Mid-Cap (2)		Small-Cap (3)		Micro-Cap (4)		Nano-Cap (5)	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
$\leq$ 80%	2.09	2.12	1.76	1.78	1.47	1.48	1.01	1.02	1.08	1.09
80-90%	2.27	2.27	1.84	1.86	0.99	0.99	1.16	1.16	0.33	0.30
90-95%	2.36	2.38	1.62	1.63	1.05	1.07	1.16	1.17	0.14	0.14
95-98%	2.12	2.13	1.81	1.83	1.59	1.60	0.71	0.71	0.07	0.06
$\geq$ 98%	2.31	2.20	1.96	1.96	1.29	1.30	0.33	0.33	0.11	0.08

Table 6: Dividend Yield (Data and Model)

This table reports dividend yields for the 25 size and fee yield groups. *Data* columns show the time-series average of groups' dividend yields. The dividend yield of group i in quarter t is computed as

$$DividendYield_t^i = \frac{\sum_j Dividend_{j,t}^i}{\sum_j MarketCapitalization_{j,t}^i}, \quad i = 1, 2, \ldots 25,$$

where $Dividend_{j,t}^i$ and $MarketCapitalization_{j,t}^i$ are dividend and market capitalization of stock j in group i, respectively. *Model* columns show the dividend yields calibrated from the model. The data of ordinary cash dividends are from CRSP. Dividend yields are annualized and in percentage. The sample period is from 2007 through 2021.

Markov Matrix across the 25 Size and Fee Yield Groups

From/To		Large-Cap					Mid-Cap					Small-Cap					Micro-Cap					Nano-Cap				
Fee Yield Percentiles		$\leq$ 80%	80-90%	90-95%	95-98%	$\geq$ 98%	$\leq$ 80%	80-90%	90-95%	95-98%	$\geq$ 98%	$\leq$ 80%	80-90%	90-95%	95-98%	$\geq$ 98%	$\leq$ 80%	80-90%	90-95%	95-98%	$\geq$ 98%	$\leq$ 80%	80-90%	90-95%	95-98%	$\geq$ 98%
Large-Cap	$\leq$ 80%	0.96	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	80-90%	0.45	0.39	0.12	0.02	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	90-95%	0.11	0.28	0.42	0.15	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	95-98%	0.06	0.07	0.23	0.49	0.13	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$\geq$ 98%	0.07	0.01	0.06	0.17	0.67	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Mid-Cap	$\leq$ 80%	0.01	0.00	0.00	0.00	0.00	0.95	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	80-90%	0.00	0.00	0.00	0.00	0.00	0.32	0.53	0.12	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	90-95%	0.00	0.00	0.00	0.00	0.00	0.07	0.28	0.49	0.13	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	95-98%	0.00	0.00	0.00	0.00	0.01	0.04	0.06	0.22	0.53	0.13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$\geq$ 98%	0.00	0.00	0.00	0.00	0.02	0.02	0.01	0.01	0.22	0.69	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Small-Cap	$\leq$ 80%	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.92	0.03	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	80-90%	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.28	0.54	0.10	0.01	0.00	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	90-95%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.06	0.27	0.45	0.12	0.02	0.03	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	95-98%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.04	0.26	0.45	0.14	0.01	0.03	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	$\geq$ 98%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.02	0.04	0.22	0.59	0.00	0.01	0.03	0.03	0.03	0.00	0.00	0.00	0.00	0.00
Micro-Cap	$\leq$ 80%	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.95	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	80-90%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.01	0.00	0.33	0.51	0.07	0.01	0.00	0.01	0.00	0.00	0.00	0.00
	90-95%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.01	0.02	0.02	0.06	0.26	0.43	0.13	0.03	0.01	0.01	0.00	0.00	0.00
	95-98%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.01	0.04	0.01	0.07	0.28	0.39	0.15	0.01	0.01	0.01	0.01	0.00
	$\geq$ 98%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.01	0.05	0.00	0.03	0.05	0.24	0.55	0.00	0.01	0.01	0.01	0.01
Nano-Cap	$\leq$ 80%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.09	0.01	0.01	0.00	0.00	0.85	0.03	0.00	0.00	0.00
	80-90%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.03	0.02	0.02	0.01	0.29	0.47	0.11	0.02	0.01
	90-95%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.02	0.04	0.02	0.07	0.30	0.39	0.11	0.03
	95-98%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.04	0.04	0.02	0.09	0.26	0.40	0.13
	$\geq$ 98%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.08	0.01	0.04	0.07	0.24	0.52

Table 7: Markov Matrix

This table reports the Markov matrix of quarterly transition rates of firms across the 25 size and fee yield groups. The quarter-t transition rate from group i to group j is computed as $p_t^{i\rightarrow j} = \frac{TotalMarketValue_t^{i\rightarrow j}}{\sum_{j=1}^{25} TotalMarketValue_t^{i\rightarrow j}}, i, j = 1, 2, \ldots, 25$, where $TotalMarketValue_t^{i\rightarrow j}$ is the sum of market capitalizations of stocks moving from group i to group j over quarter t. The Markov matrix presents the time-series average of transition rates, which are standardized such that the sum of rates in each row equals one.

Panel A. CAPM Risk Premia (annual, in percent)

Fee Yield Percentiles	Large-Cap (1)	Mid-Cap (2)	Small-Cap (3)	Micro-Cap (4)	Nano-Cap (5)
$\leq$ 80%	6.57	7.65	8.12	8.01	7.60
80-90%	7.34	8.44	9.44	10.25	8.68
90-95%	7.93	9.18	10.18	12.40	10.26
95-98%	9.18	10.70	10.16	10.89	8.92
$\geq$ 98%	9.87	11.63	9.97	11.75	10.40

Panel B. Carhart 4-Factor Risk Premia (annual, in percent)

Fee Yield Percentiles	Large-Cap (1)	Mid-Cap (2)	Small-Cap (3)	Micro-Cap (4)	Nano-Cap (5)
$\leq$ 80%	6.47	7.73	9.74	7.19	6.73
80-90%	6.78	7.99	9.92	9.65	7.66
90-95%	7.45	9.14	11.53	9.45	9.79
95-98%	6.51	8.88	11.29	8.84	4.69
$\geq$ 98%	6.94	9.90	11.46	9.43	8.27

Table 8: Expected Risk Premium

This table reports estimated risk premia for the 25 size and fee yield groups. The risk premium of a group is computed as factor premia times the group's factor loadings, which are estimated from regressions of monthly value-weighted returns on factor returns over the sample period 2007–2021. The factor premia are estimated as the time-series average of factor returns from 1972 to 2021. We choose this long time period to estimate risk premia because it covers major business cycles over the past decades and CRSP includes NASDAQ stocks starting from 1972. Panel A shows the risk premia of the 25 groups estimated from CAPM, and Panel B shows the risk premia estimated from Carhart 4-factor model. Risk premia are annualized and reported in percent.

Panel A. Value Wedges due to Lending Fee Income (in percent)

Fee Yield Percentiles	Large-Cap (1)	Mid-Cap (2)	Small-Cap (3)	Micro-Cap (4)	Nano-Cap (5)
$\leq$ 80%	1.50	2.78	6.34	7.81	16.91
80-90%	1.60	3.30	7.92	12.31	34.71
90-95%	1.76	4.17	10.95	21.97	54.36
95-98%	2.03	5.85	16.49	33.83	76.55
$\geq$ 98%	2.61	9.17	25.35	49.28	100.08

Panel B. Value Wedges under Pro-rata Allocation of Fee Income (in percent)

Fee Yield Percentiles	Large-Cap (1)	Mid-Cap (2)	Small-Cap (3)	Micro-Cap (4)	Nano-Cap (5)
$\leq$ 80%	0.36	0.68	1.29	1.39	2.35
80-90%	0.39	0.80	1.58	2.05	3.83
90-95%	0.42	0.99	2.12	3.34	5.40
95-98%	0.48	1.37	3.13	4.66	6.93
$\geq$ 98%	0.61	2.06	4.68	6.26	8.61

Table 9: Value Wedges
The table reports value wedges for the 25 size and fee yield groups. Panel A reports the incremental value shares lenders assign to stocks of a group because of the lending fee income that is generated by the stocks. Panel B reports value wedges under a counterfactual pro-rata allocation of lending fee income across all shares (rather than just the shares available for lending).

Panel A. Distribution of Aggregate Value Added from Fee Income

Fee Yield Percentiles	Large-Cap (1)	Mid-Cap (2)	Small-Cap (3)	Micro-Cap (4)	Nano-Cap (5)
$\leq$ 80%	38.83	25.51	11.27	15.47	0.88
80-90%	1.21	1.16	0.88	0.41	0.04
90-95%	0.52	0.58	0.47	0.27	0.02
95-98%	0.30	0.39	0.36	0.18	0.02
$\geq$ 98%	0.27	0.43	0.32	0.18	0.02

Panel B. Distribution of Aggregate Value Added under Pro-rata Allocation of Fee Income

Fee Yield Percentiles	Large-Cap (1)	Mid-Cap (2)	Small-Cap (3)	Micro-Cap (4)	Nano-Cap (5)
$\leq$ 80%	29.02	21.44	9.95	21.00	0.59
80-90%	2.11	2.41	1.49	1.55	0.11
90-95%	1.12	1.32	0.90	0.74	0.07
95-98%	0.73	1.00	0.78	0.60	0.06
$\geq$ 98%	0.71	0.95	0.78	0.52	0.05

Table 10: Distribution across Size and Fee Yield Groups of Total Value Added from Lending Fee Income

This table reports the percentage of total value added from lending fee income across the 25 size and fee yield groups. The reported percentage numbers are computed as the time-series averages of the percentages for each group. In panel A, the value added of a group is computed as its value wedge times the total market value of lenders' inventory for stocks in this group. In panel B, the value added of a group is computed as the value wedge under a counterfactual pro-rata allocation of lending fee income times the total market value of stocks in this group. All numbers are reported in percent.

Short Sellers' Alphas (annual, in percent)

Fee Yield Percentiles	Large-Cap (1)	Mid-Cap (2)	Small-Cap (3)	Micro-Cap (4)	Nano-Cap (5)
$\leq$ 80%	0.10	0.08	0.13	0.11	0.26
80-90%	0.19	0.23	0.39	1.49	16.11
90-95%	0.21	0.38	0.94	4.58	27.55
95-98%	0.24	1.03	2.40	10.67	39.67
$\geq$ 98%	1.29	4.77	8.85	29.30	60.00

Table 11: Short Sellers' Alphas
This table reports the model-implied expected excess returns short sellers obtain net of fees in each group i per annum (see equation (30). All numbers are annualized and reported in percent.

Volatilities of Informed Traders' Signals (annual, in percent)

Fee Yield Percentiles	Large-Cap (1)	Mid-Cap (2)	Small-Cap (3)	Micro-Cap (4)	Nano-Cap (5)
$\leq$ 80%	0.05	0.05	0.07	0.06	0.14
80-90%	0.10	0.13	0.22	0.85	9.24
90-95%	0.11	0.21	0.54	2.68	16.11
95-98%	0.13	0.59	1.39	6.32	23.51
$\geq$ 98%	0.74	2.82	5.26	17.53	36.12

Table 12: Estimated Volatilities of Informed Traders' Signals
This table reports the estimated volatilities of informed traders' signals in each group i. Specifically, we compute for each state s:

$$(\lambda \cdot (e^{\sigma_v(s)^2} - 1))^{1/2}.$$

All numbers are annualized and reported in percent.

Idiosyncratic Volatility (annual, in percent)

Fee Yield Percentiles	Large-Cap (1)	Mid-Cap (2)	Small-Cap (3)	Micro-Cap (4)	Nano-Cap (5)
≤ 80%	19.75	25.52	32.99	26.88	59.57
80-90%	23.13	30.75	43.25	42.57	93.27
90-95%	24.41	34.78	47.39	51.93	108.54
95-98%	26.51	39.22	48.66	64.97	128.20
≥ 98%	27.61	42.62	61.11	73.77	135.27

Table 13: Idiosyncratic Volatility

This table reports the time-series average of value-weighted stock idiosyncratic volatilities across the 25 size and fee yield groups, where the weight is market capitalization. The idiosyncratic volatility of a stock is estimated as the standard deviation of residuals from regressions of the stock's daily returns on the CRSP value-weighted market returns over the one-year rolling window. All numbers are annualized and reported in percent.

E Robustness Analyses

Panel A. Removing Observations Around Voting Record Dates

	Lending Fee (annual, in percent)				
	Large-Cap	Mid-Cap	Small-Cap	Micro-Cap	Nano-Cap
Fee Yield Percentiles	(1)	(2)	(3)	(4)	(5)
$\leq 80\%$	0.261	0.252	0.264	0.491	3.897
80-90%	0.263	0.281	0.452	2.125	17.067
90-95%	0.266	0.435	1.109	5.992	29.834
95-98%	0.291	1.144	2.844	13.473	46.027
$\geq 98\%$	2.061	6.263	11.380	37.578	73.728

Panel B. Removing Observations Around Ex-Dividend Dates

	Lending Fee (annual, in percent)				
	Large-Cap	Mid-Cap	Small-Cap	Micro-Cap	Nano-Cap
Fee Yield Percentiles	(1)	(2)	(3)	(4)	(5)
$\leq 80\%$	0.206	0.211	0.245	0.411	3.913
80-90%	0.214	0.253	0.438	2.075	18.113
90-95%	0.222	0.396	1.064	6.057	31.060
95-98%	0.245	1.040	2.779	13.985	47.841
$\geq 98\%$	1.860	6.172	11.583	39.177	74.410

Table 14: Lending Fee
This table reports the time-series average of lending fees for the 25 size and fee yield groups. In panel A, we remove observations 15 days before and after voting record dates. Following Aggarwal et al. (2015), we collect the data of voting record dates from ISS. In panel B, we remove observations 15 days before and after dividend ex-dividend dates. The lending fee of group i in quarter t is computed as

$$LendingFee_t^i = \frac{\sum_j ValueOnLoan_{j,t}^i \times LendingFee_{i,t}^i}{\sum_j ValueOnLoan_{j,t}^i}, \quad i = 1, 2, \ldots 25,$$

where $LendingFee_{j,t}^i$ and $ValueOnLoan_{j,t}^i$ are lending fees and value of shares on loan for stock j in group i, respectively. Lending fees are annualized and in percentage. The sample period is from 2007 through 2021.